

4-6

Triangle Congruence: ASA, AAS, and HL

Warm Up

Lesson Presentation

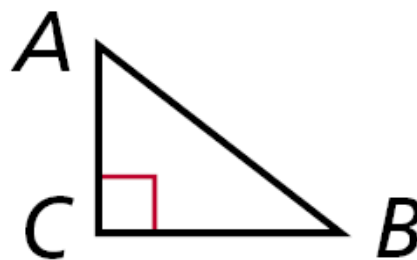
Lesson Quiz

4-6 Triangle Congruence: ASA, AAS, and HL

Warm Up

1. What are sides AC and BC called? Side AB ?

legs; hypotenuse



2. Which side is in between $\angle A$ and $\angle C$?

\overline{AC}

3. Given $\triangle DEF$ and $\triangle GHI$, if $\angle D \cong \angle G$ and $\angle E \cong \angle H$, why is $\angle F \cong \angle I$?

Third \angle s Thm.

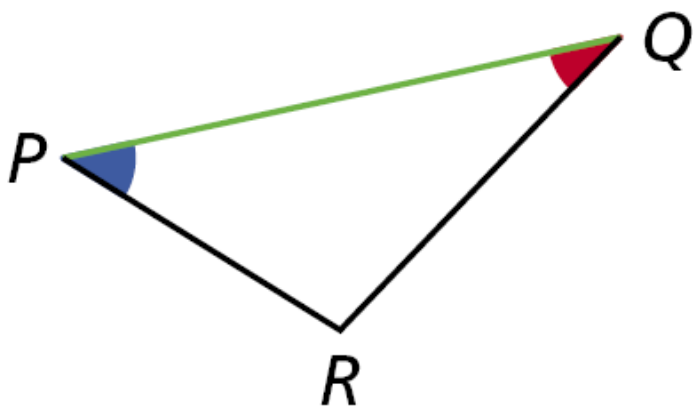
Objectives

Apply ASA, AAS, and HL to construct triangles and to solve problems.

Prove triangles congruent by using ASA, AAS, and HL.

4-6 Triangle Congruence: ASA, AAS, and HL

An **included side** is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an included side.



\overline{PQ} is the included side of $\angle P$ and $\angle Q$.

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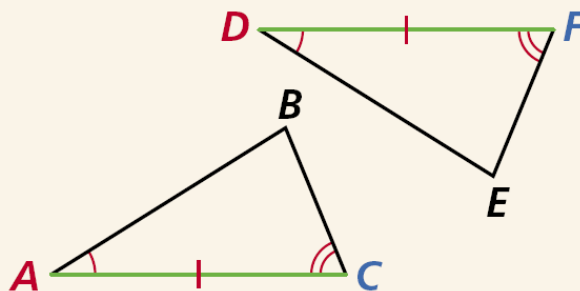
Postulate 4-5-1

Angle-Side-Angle (ASA) Congruence

POSTULATE

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

HYPOTHESIS

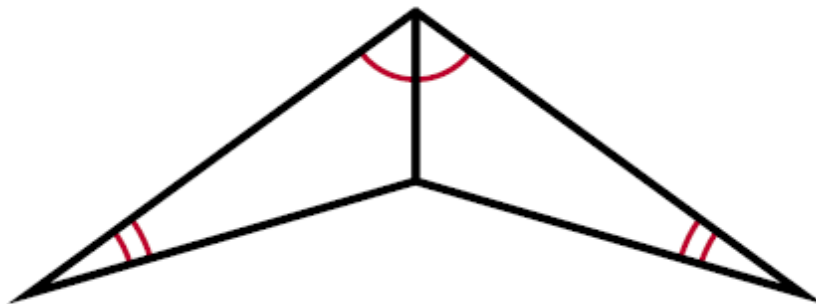


CONCLUSION

$$\triangle ABC \cong \triangle DEF$$

Example 2: Applying ASA Congruence

Determine if you can use ASA to prove the triangles congruent. Explain.



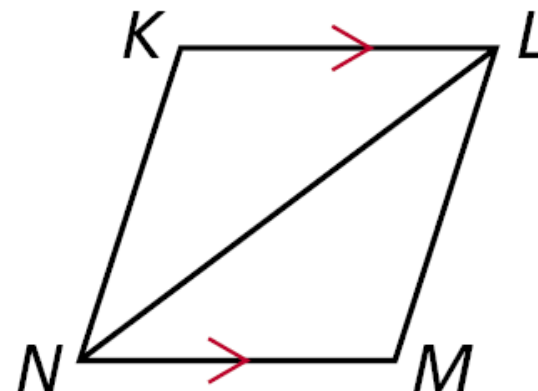
Two congruent angle pairs are given, but the included sides are not given as congruent. Therefore ASA cannot be used to prove the triangles congruent.

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Triangle Congruence: ASA, AAS, and HL

Check It Out! Example 2

Determine if you can use ASA to prove $\triangle NKL \cong \triangle LMN$. Explain.



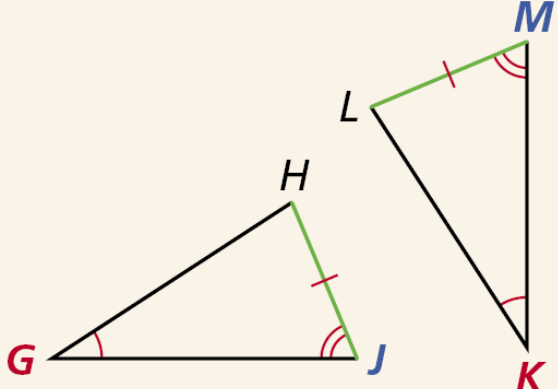
By the Alternate Interior Angles Theorem. $\angle KLN \cong \angle MNL$. $\overline{NL} \cong \overline{LN}$ by the Reflexive Property. No other congruence relationships can be determined, so ASA cannot be applied.

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You can use the Third Angles Theorem to prove another congruence relationship based on ASA. This theorem is Angle-Angle-Side (AAS).

Theorem 4-5-2

Angle-Angle-Side (AAS) Congruence

THEOREM	HYPOTHESIS	CONCLUSION
If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.		$\triangle GHJ \cong \triangle KLM$

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Triangle Congruence: ASA, AAS, and HL

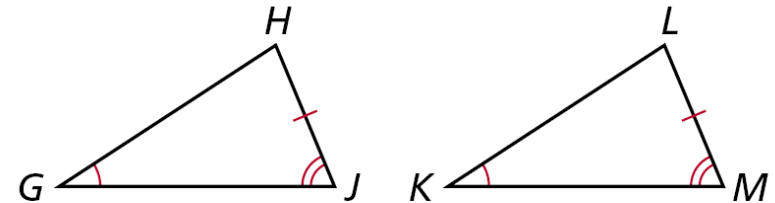
PROOF

Angle-Angle-Side Congruence

Given: $\angle G \cong \angle K$, $\angle J \cong \angle M$, $\overline{HJ} \cong \overline{LM}$

Prove: $\triangle GHJ \cong \triangle KLM$

Proof:



Statements	Reasons
1. $\angle G \cong \angle K$, $\angle J \cong \angle M$	1. Given
2. $\angle H \cong \angle L$	2. Third \angle Thm.
3. $\overline{HJ} \cong \overline{LM}$	3. Given
4. $\triangle GHJ \cong \triangle KLM$	4. ASA <i>Steps 1, 3, and 2</i>

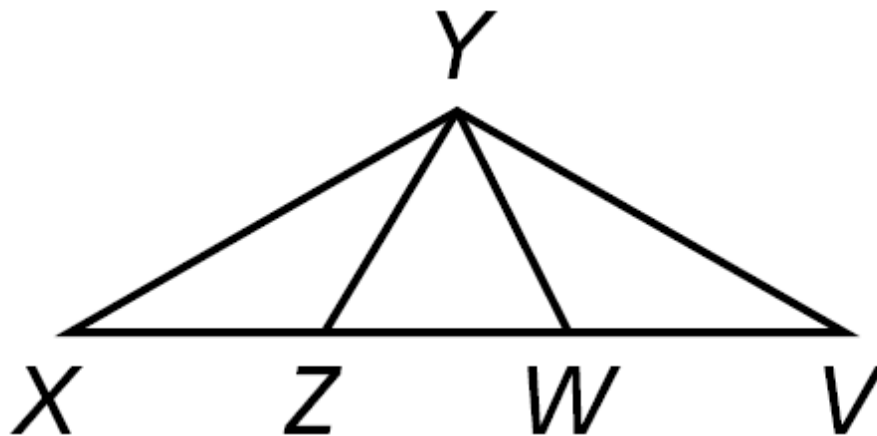
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Example 3: Using AAS to Prove Triangles Congruent

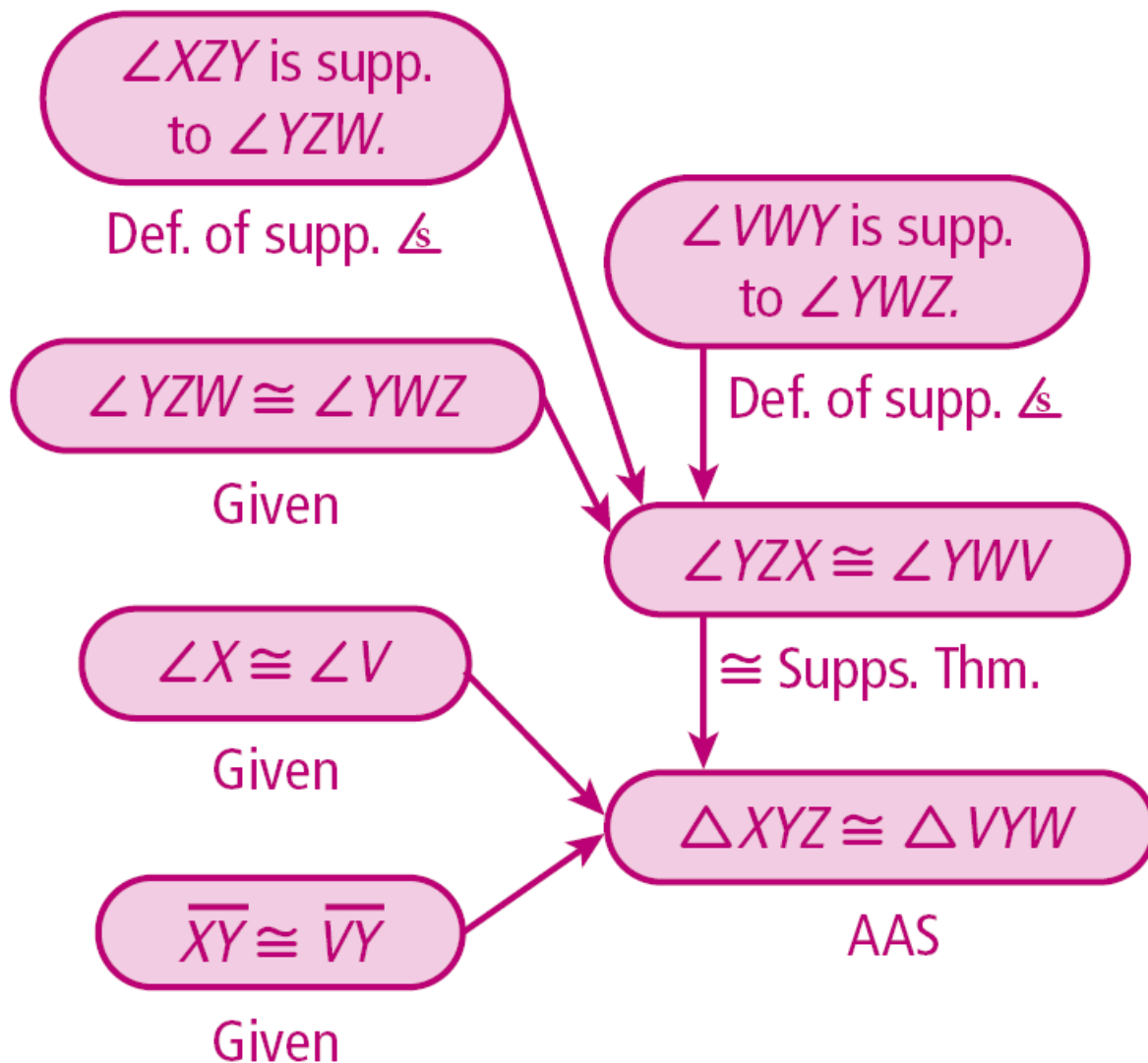
Use AAS to prove the triangles congruent.

Given: $\angle X \cong \angle V$, $\angle YZW \cong \angle YWZ$, $\overline{XY} \cong \overline{VY}$

Prove: $\triangle XYZ \cong \triangle VYW$



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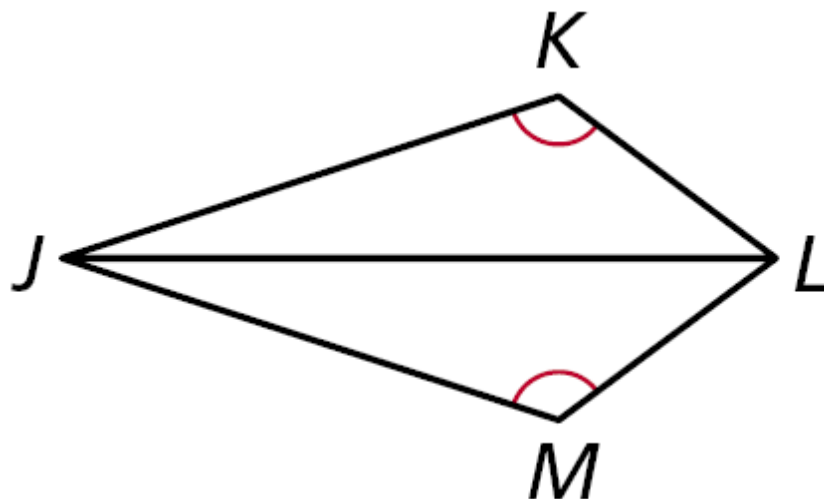
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Check It Out! Example 3

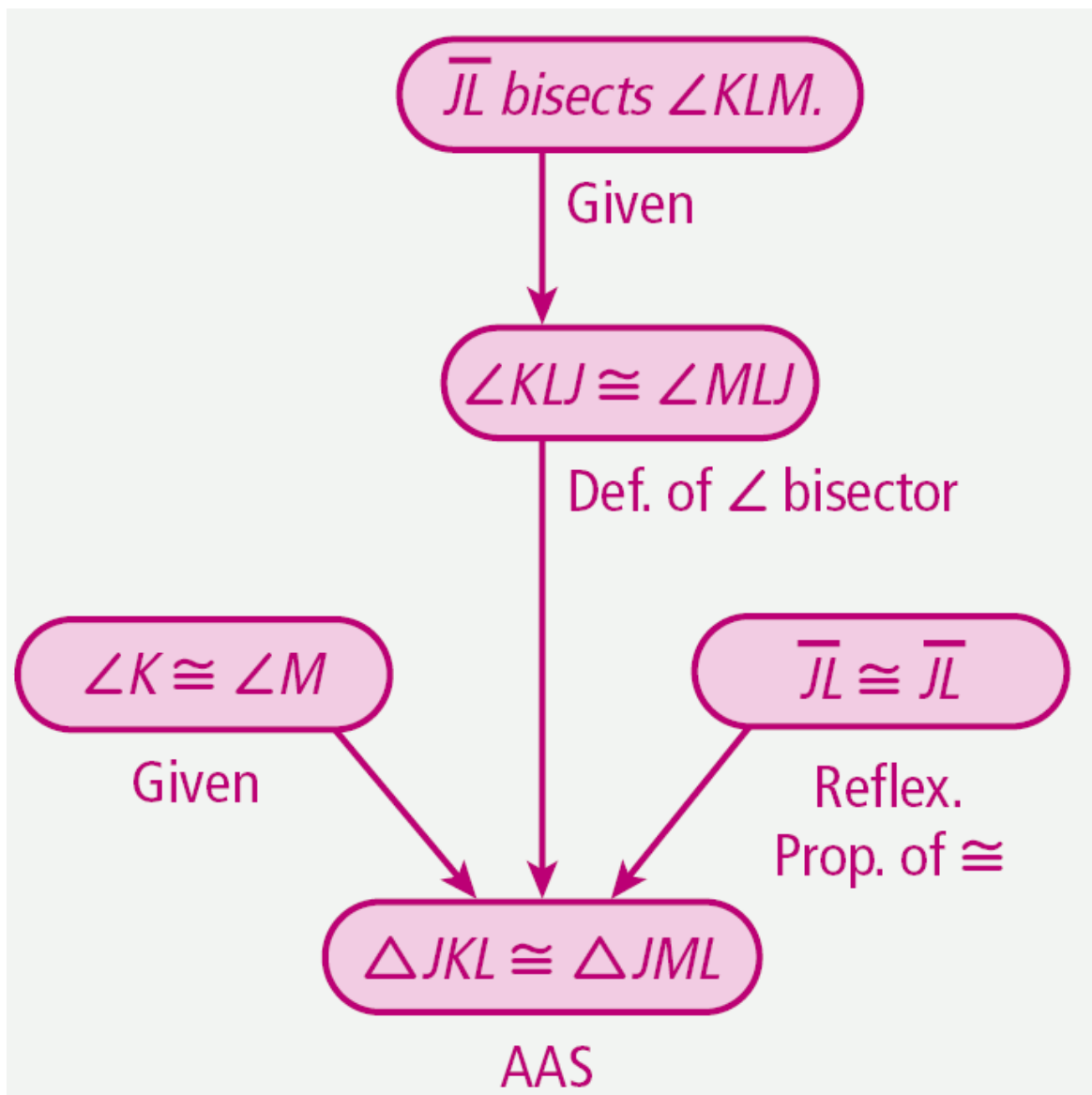
Use AAS to prove the triangles congruent.

Given: \overline{JL} bisects $\angle KLM$, $\angle K \cong \angle M$

Prove: $\triangle JKL \cong \triangle JML$



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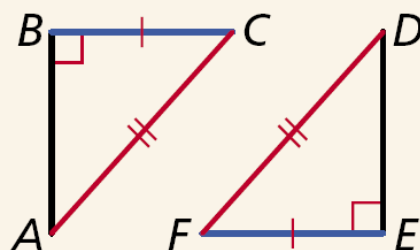


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Triangle Congruence: ASA, AAS, and HL

Theorem 4-5-3**Hypotenuse-Leg (HL) Congruence****THEOREM**

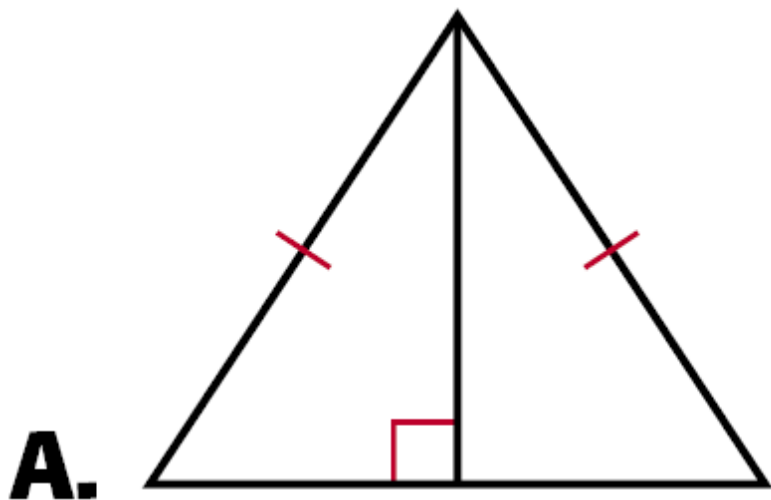
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

HYPOTHESIS**CONCLUSION**

$$\triangle ABC \cong \triangle DEF$$

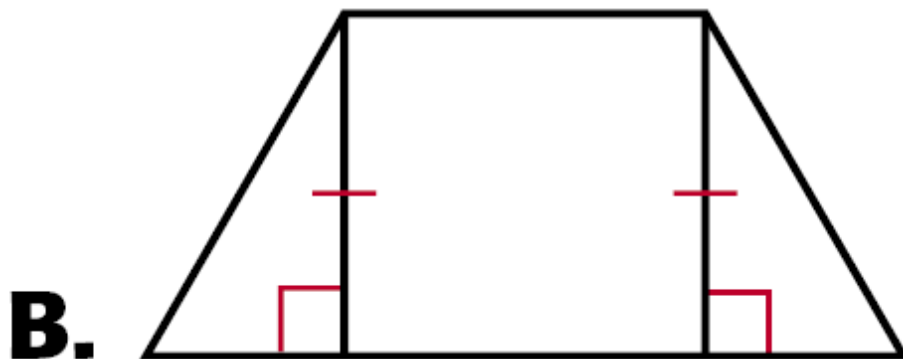
Example 4A: Applying HL Congruence

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.



According to the diagram, the triangles are right triangles that share one leg.

It is given that the hypotenuses are congruent, therefore the triangles are congruent by HL.

Example 4B: Applying HL Congruence

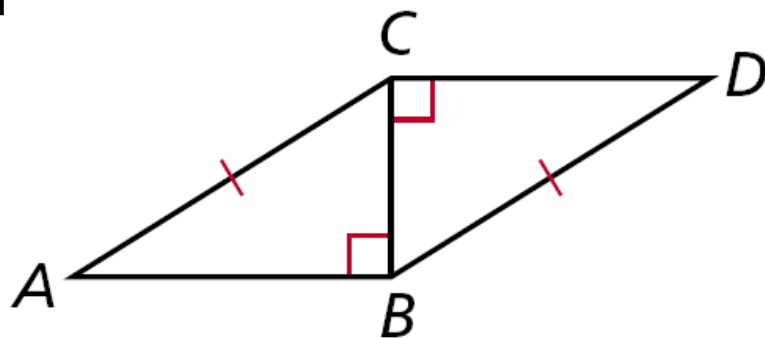
This conclusion cannot be proved by HL. According to the diagram, the triangles are right triangles and one pair of legs is congruent. You do not know that one hypotenuse is congruent to the other.

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Check It Out! Example 4

Determine if you can use the HL Congruence Theorem to prove $\triangle ABC \cong \triangle DCB$. If not, tell what else you need to know.



Yes; it is given that $\overline{AC} \cong \overline{DB}$. $\overline{BC} \cong \overline{CB}$ by the Reflexive Property of Congruence. Since $\angle ABC$ and $\angle DCB$ are right angles, $\triangle ABC$ and $\triangle DCB$ are right triangles. $\triangle ABC \cong \triangle DCB$ by *HL*.