13-2 Theoretical and Experimental Probability

Warm Up

Lesson Presentation

Lesson Quiz

Objectives

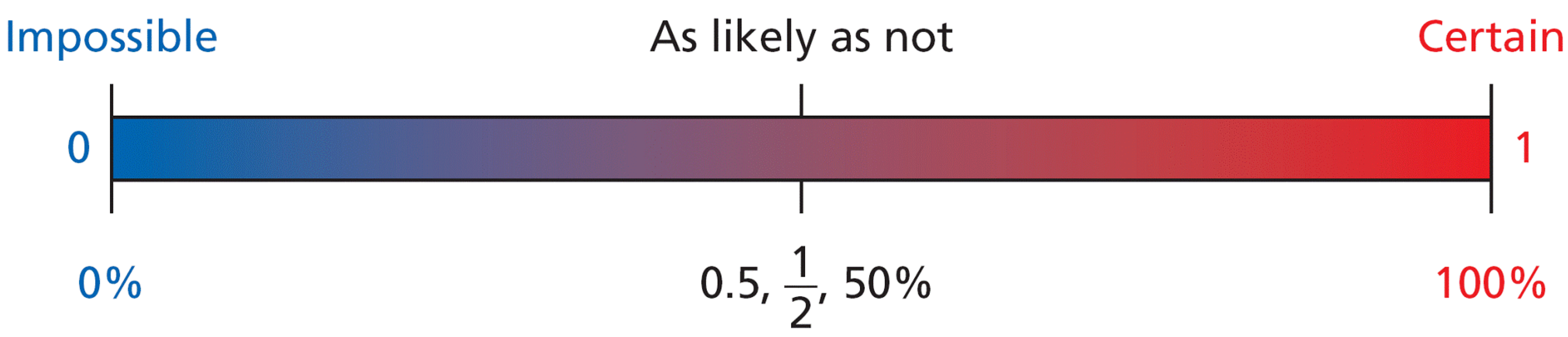
Find the theoretical probability of an event.

Find the experimental probability of an event.

Probability is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome**. The **sample space** is the set of all possible outcomes. An **event** is an outcome or set of outcomes.

Experiment or Situation	Rolling a number cube 	Spinning a spinner 
Sample Space	{1, 2, 3, 4, 5, 6}	{red, blue, green, yellow}

Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.



Equally likely outcomes have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Theoretical Probability

For equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}.$$

Example 1A: Finding Theoretical Probability

Each letter of the word PROBABLE is written on a separate card. The cards are placed face down and mixed up. What is the probability that a randomly selected card has a consonant?

There are 8 possible outcomes and 5 favorable outcomes.

$$P(\text{consonant}) = \frac{5}{8} = 62.5\%$$

Example 1B: Finding Theoretical Probability

Two number cubes are rolled. What is the probability that the difference between the two numbers is 4?

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

There are 36 possible outcomes.

$$P(\text{difference is } 4) = \frac{\text{number of outcomes with a difference of 4}}{36}$$

$$P(\text{difference is } 4) = \frac{4}{36} = \frac{1}{9}$$

4 outcomes with a difference of 4: (1, 5), (2, 6), (5, 1), and (6, 2)

Check It Out! Example 1a

A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event?

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

The sum is 6.

There are 36 possible outcomes.

$$P(\text{sum is 6}) = \frac{\text{number of outcomes with a sum of 6}}{36}$$

$$P(\text{sum is 6}) = \frac{5}{36}$$

5 outcomes with a sum of 6: (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1)

Check It Out! Example 1b

A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event?

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

The difference is 6.

There are 36 possible outcomes.

$$P(\text{difference is 6}) = \frac{\text{number of outcomes with a difference of 6}}{36}$$

$$P(\text{difference is 6}) = \frac{0}{36}$$

0 outcomes with a difference of 6

Check It Out! Example 1c

A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of the event?

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

The red cube is greater.

There are 36 possible outcomes.

$$P(\text{red cube greater}) = \frac{\text{number of outcomes with the red cube greater}}{36}$$

15 outcomes with a red greater than blue: (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (3, 2), (4, 2), (5, 2), (6, 2), (4, 3), (5, 3), (6, 3), (5, 4), (6, 4) and (6, 5).

$$P(\text{red cube greater}) = \frac{15}{36} = \frac{5}{12}$$

The sum of all probabilities in the sample space is 1. The **complement** of an event E is the set of all outcomes in the sample space that are not in E.

Complement

The probability of the complement of event E is

$$P(\text{not } E) = 1 - P(E).$$

Example 2: Application

There are 25 students in study hall. The table shows the number of students who are studying a foreign language. What is the probability that a randomly selected student is not studying a foreign language?

Language	Number
French	6
Spanish	12
Japanese	3

Example 2 Continued

P(not foreign) = 1 – P(foreign) *Use the complement.*

$$P(\text{not foreign}) = 1 - \frac{21}{25}$$

There are 21 students studying a foreign language.

$$= \frac{4}{25}, \text{ or } 16\%$$

There is a 16% chance that the selected student is not studying a foreign language.

Check It Out! Example 2

Two integers from 1 to 10 are randomly selected. The same number may be chosen twice. What is the probability that both numbers are less than 9?

$P(\text{number} < 9) = 1 - P(\text{number} \geq 9)$ *Use the complement.*

$$P(\text{number} < 9) = 1 - \frac{2}{10} = \frac{8}{10}$$

The probability that both numbers are less than 9, is

$$\frac{8}{10} \cdot \frac{8}{10} = \frac{64}{100} = \frac{16}{25}, \text{ or } 64\%.$$

Example 3: Finding Probability with Permutations or Combinations

Each student receives a 5-digit locker combination. What is the probability of receiving a combination with all odd digits?

Step 1 Determine whether the code is a permutation or a combination.

Order is important, so it is a permutation.

Check It Out! Example 3

A DJ randomly selects 2 of 8 ads to play before her show. Two of the ads are by a local retailer. What is the probability that she will play both of the retailer's ads before her show?

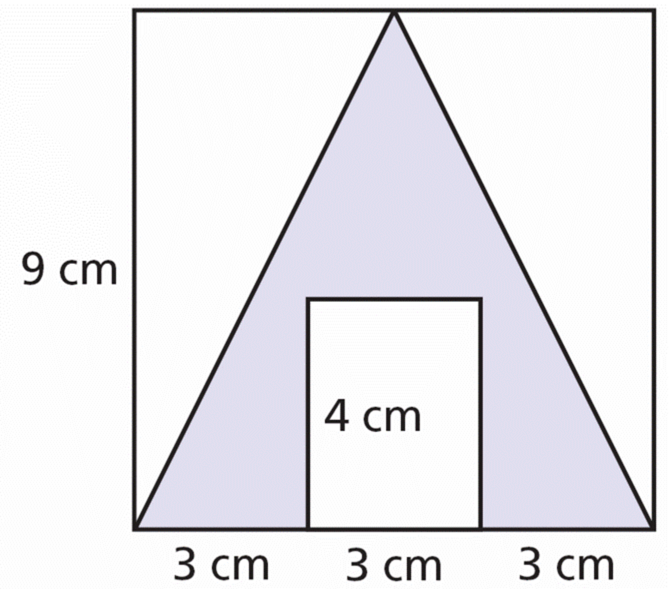
Step 1 Determine whether the code is a permutation or a combination.

Order is not important, so it is a combination.

Geometric probability is a form of theoretical probability determined by a ratio of lengths, areas, or volumes.

Example 4: Finding Geometric Probability

A figure is created placing a rectangle inside a triangle inside a square as shown. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



Example 4 Continued

Find the ratio of the area of the shaded region to the area of the entire square. The area of a square is s^2, the area of a triangle is $\frac{1}{2}bh$, and the area of a rectangle is lw.

First, find the area of the entire square.

$A_t = (9)^2 = 81$ *Total area of the square.*

Example 4 Continued

Next, find the area of the triangle.

$$A_{\text{triangle}} = \frac{1}{2}(9)(9) = 40.5$$ *Area of the triangle.*

Next, find the area of the rectangle.

$$A_{\text{rectangle}} = (3)(4) = 12$$ *Area of the rectangle.*

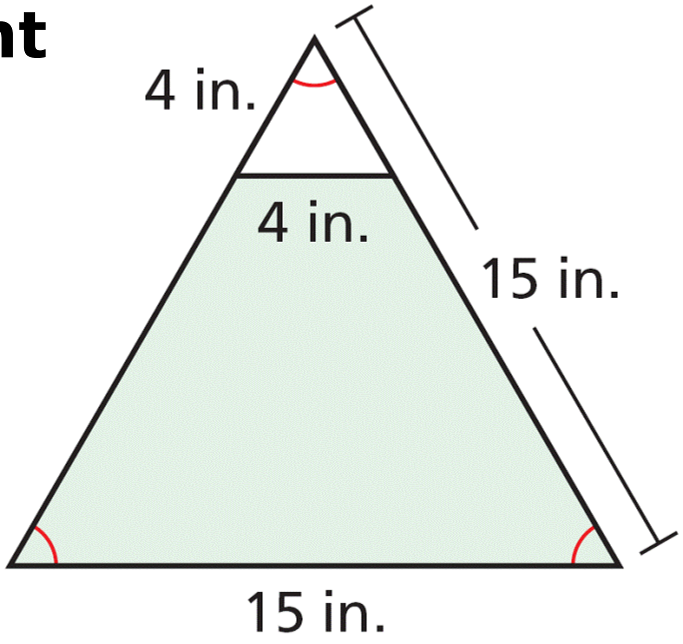
Subtract to find the shaded area.

$$A_s = 40.5 - 12 = 28.5$$ *Area of the shaded region.*

$$\frac{A_s}{A_t} = \frac{28.5}{81} = \frac{19}{54} \approx 0.352$$ *Ratio of the shaded region to total area.*

Check It Out! Example 4

Find the probability that a point chosen at random inside the large triangle is in the small triangle.



The probability that a point is inside the small triangle is the ratio of the area of small triangle to the large triangle.

The area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$, where s is the side.

Check It Out! Example 4 Continued

First, find the area of the small triangle.

$$A_{small} = \frac{s^2\sqrt{3}}{4} = \frac{4^2\sqrt{3}}{4} = \frac{16\sqrt{3}}{4} = 4\sqrt{3}$$ *Area of the small triangle.*

Next, find the area of the large triangle.

$$A_{large} = \frac{s^2\sqrt{3}}{4} = \frac{15^2\sqrt{3}}{4} = \frac{225\sqrt{3}}{4}$$ *Area of the large triangle.*

$$\frac{A_{small}}{A_{large}} = \frac{4\sqrt{3}}{\frac{225\sqrt{3}}{4}} = \frac{4\sqrt{3}}{1} \bullet \frac{4}{225\sqrt{3}} = \frac{16\sqrt{3}}{225\sqrt{3}} = \frac{16}{225}$$ *Ratio of the small triangle to the large triangle.*

You can estimate the probability of an event by using data, or by **experiment**. For example, if a doctor states that an operation "has an 80% probability of success," 80% is an estimate of probability based on similar case histories.

Each repetition of an experiment is a **trial**. The sample space of an experiment is the set of all possible outcomes. The **experimental probability** of an event is the ratio of the number of times that the event occurs, the *frequency*, to the number of trials.

Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Experimental probability is often used to estimate theoretical probability and to make predictions.

Example 5A: Finding Experimental Probability

The table shows the results of a spinner experiment. Find the experimental probability.

Number	Occurrences
1	6
2	11
3	19
4	14

spinning a 4

The outcome of 4 occurred 14 times out of 50 trials.

$$P(4) = \frac{14}{50} = \frac{7}{25} = 0.28$$

Example 5B: Finding Experimental Probability

The table shows the results of a spinner experiment. Find the experimental probability.

Number	Occurrences
1	6
2	11
3	19
4	14

spinning a number greater than 2

The numbers 3 and 4 are greater than 2.

$$P(\text{greater than } 2) = \frac{19+14}{50} = \frac{33}{50} = 0.66$$

3 occurred 19 times and 4 occurred 14 times.

Check It Out! Example 5a

The table shows the results of choosing one card from a deck of cards, recording the suit, and then replacing the card.

Card Suit	Hearts	Diamonds	Clubs	Spades
Number	5	9	7	5

Find the experimental probability of choosing a diamond.

The outcome of diamonds occurred 9 of 26 times.

$$P(\text{diamonds}) = \frac{9}{26}$$

Check It Out! Example 5b

The table shows the results of choosing one card from a deck of cards, recording the suit, and then replacing the card.

Card Suit	Hearts	Diamonds	Clubs	Spades
Number	5	9	7	5

Find the experimental probability of choosing a card that is not a club.

Use the complement.

$$P(\text{club}) = \frac{7}{26}$$

$$1 - P(\text{club}) = 1 - \frac{7}{26} = \frac{19}{26}$$