12-5 Angle Relationships in Circles

Warm Up

Lesson Presentation

Lesson Quiz

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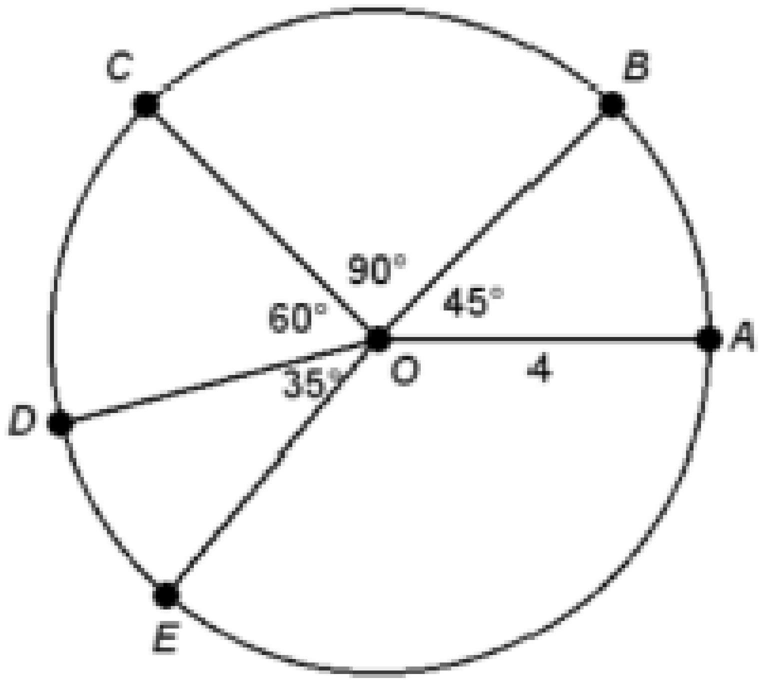
Consider the diagram below.



Find the sector area of $\widehat{CDE}$.

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Objectives

Find the measures of angles formed by lines that intersect circles.

Use angle measures to solve problems.

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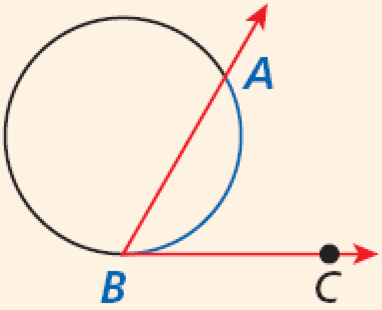
Theorem 11-5-1

THEOREM	HYPOTHESIS	CONCLUSION
If a tangent and a secant (or chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.	 Tangent $\overrightarrow{BC}$ and secant $\overrightarrow{BA}$ intersect at B.	$m\angle ABC = \frac{1}{2}m\widehat{AB}$

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Example 1A: Using Tangent-Secant and Tangent-Chord Angles

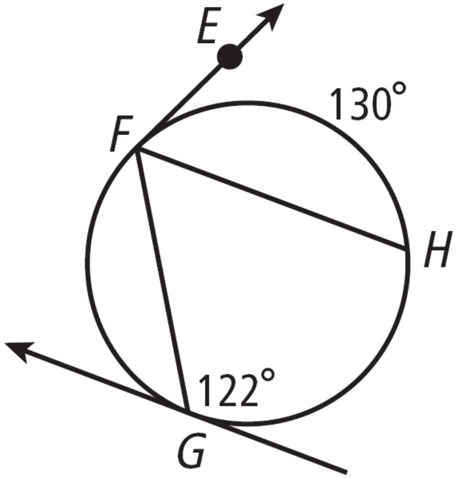
Find each measure.

m∠*EFH*

$$m\angle EFH = \frac{1}{2} m\overset{\frown}{FH}$$

$$m\angle EFH = \frac{1}{2}(130^\circ)$$

$$= 65^\circ$$



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Example 1B: Using Tangent-Secant and Tangent-Chord Angles

Find each measure.

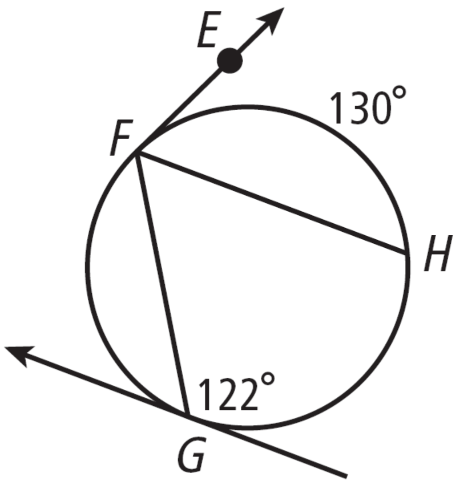
$m\overset{\frown}{GF}$

$$m\angle G = \frac{1}{2}m\overset{\frown}{GF}$$

$$180^\circ - 122^\circ = \frac{1}{2}m\overset{\frown}{GF}$$

$$58^\circ = \frac{1}{2}m\overset{\frown}{GF}$$

$$116^\circ = m\overset{\frown}{GF}$$



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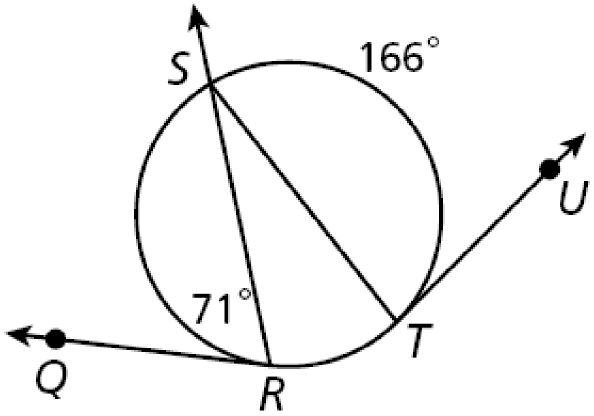
Check It Out! Example 1a

Find each measure.

m∠*STU*



$$m\angle STU = \frac{1}{2} m\overset{\frown}{ST}$$

$$m\angle STU = \frac{1}{2}(166^\circ)$$

$$= 83^\circ$$

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Check It Out! Example 1b

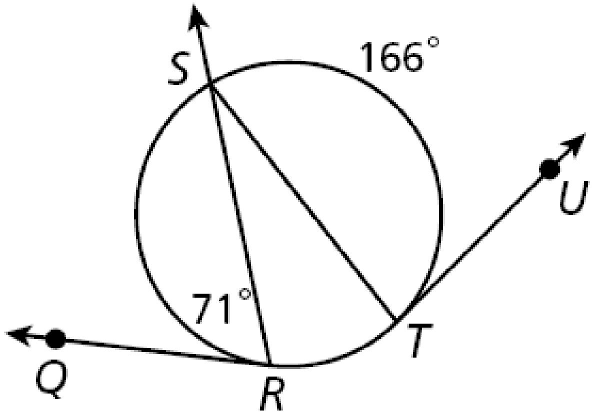
Find each measure.

m$\overset{\frown}{SR}$

$$m\angle SRQ = \frac{1}{2}m\overset{\frown}{SR}$$

$$(71°) = \frac{1}{2}\left(m\overset{\frown}{SR}\right)$$

$$142° = m\overset{\frown}{SR}$$



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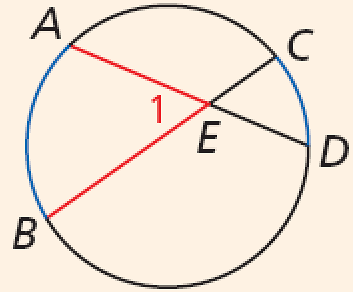
Theorem 11-5-2

THEOREM	HYPOTHESIS	CONCLUSION
If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of its intercepted arcs.	 Chords $\overline{AD}$ and $\overline{BC}$ intersect at E.	$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$

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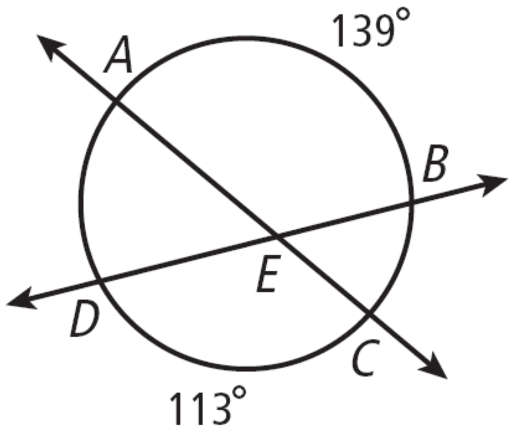
Example 2: Finding Angle Measures Inside a Circle

Find each measure.

m∠*AEB*



$$m\angle AEB = \frac{1}{2}\left(m\overparen{AB} + m\overparen{CD}\right)$$

$$= \frac{1}{2}(139^\circ + 113^\circ)$$

$$= \frac{1}{2}(252^\circ)$$

$$= 126^\circ$$

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Check It Out! Example 2a

Find each angle measure.

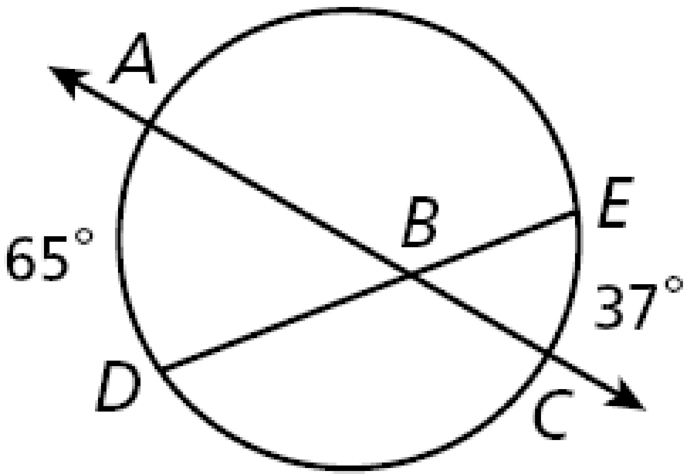
m∠*ABD*

$$m\angle ABD = \frac{1}{2}\left(m\overset{\frown}{EC} + m\overset{\frown}{AD}\right)$$

$$m\angle ABD = \frac{1}{2}\left(37^\circ + 65^\circ\right)$$

$$m\angle ABD = \frac{1}{2}\left(102^\circ\right)$$

$$m\angle ABD = 51^\circ$$



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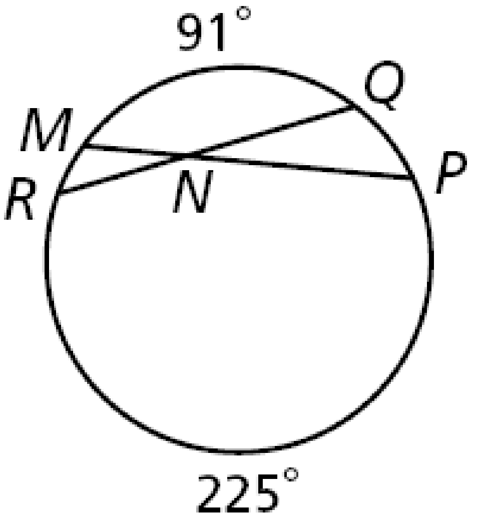
Check It Out! Example 2b

Find each angle measure.

m∠*RNM*



$$m\angle MNQ = \frac{1}{2}\left(m\overset{\frown}{MQ} + m\overset{\frown}{RP}\right)$$

$$m\angle MNQ = \frac{1}{2}(91° + 225°) = 158°$$

$$m\angle RNM = 180° - \angle MNQ$$

$$m\angle RNM = 180° - 158° = 22°$$

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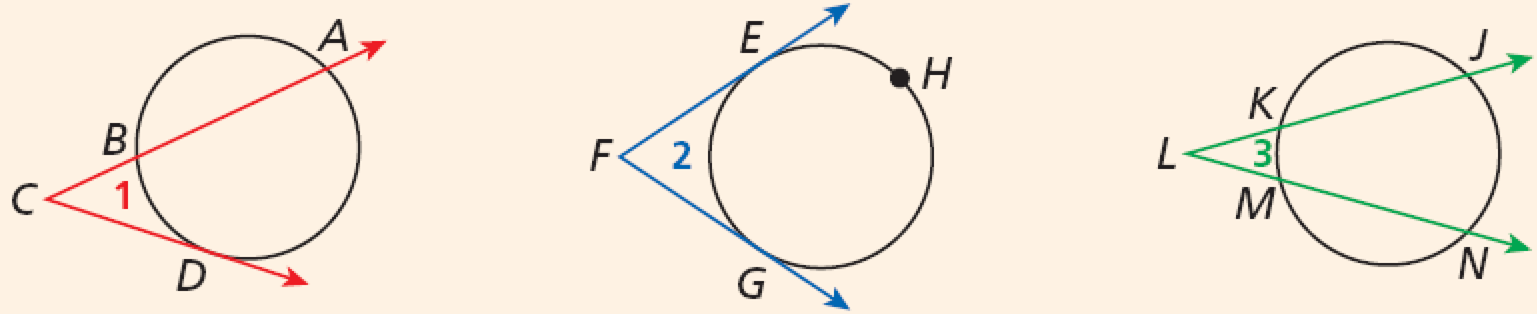
Theorem 11-5-3

If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AD} - m\overset{\frown}{BD})$ $\quad m\angle 2 = \frac{1}{2}(m\overset{\frown}{EHG} - m\overset{\frown}{EG})$ $\quad m\angle 3 = \frac{1}{2}(m\overset{\frown}{JN} - m\overset{\frown}{KM})$

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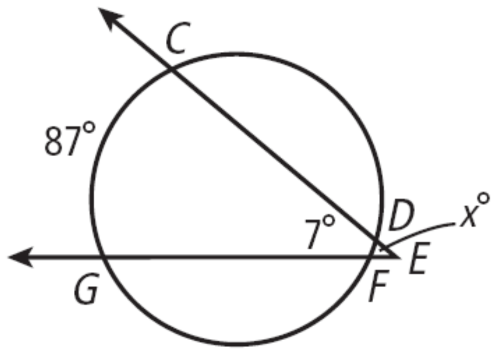
Example 3: Finding Measures Using Tangents and Secants

Find the value of *x*.

A.

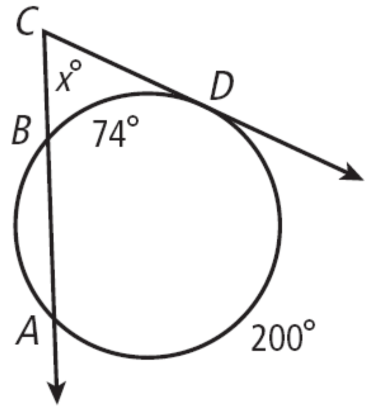


$$x = \frac{1}{2}\left(m\overset{\frown}{CG} - m\overset{\frown}{DF}\right)$$

$$= \frac{1}{2}\left(87° - 7°\right)$$

$$= 40°$$

B.



$$x = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{BD}\right)$$

$$= \frac{1}{2}\left(200° - 74°\right)$$

$$= 63°$$

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Check It Out! Example 3

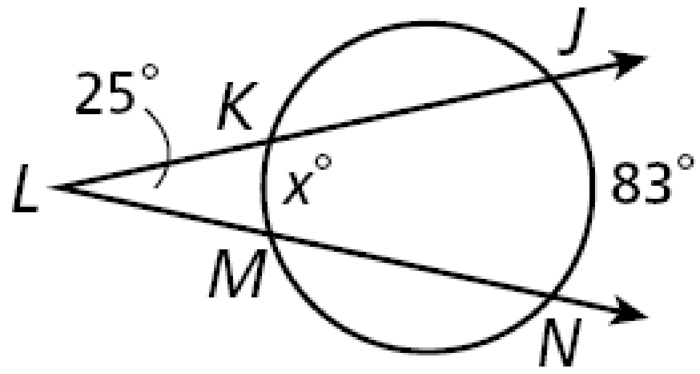
Find the value of *x*.

$$m\angle L = \frac{1}{2}\left(m\overset{\frown}{JN} - m\overset{\frown}{KM}\right)$$

$$25° = \frac{1}{2}(83° - x°)$$

$$50° = 83° - x$$

$$x = 33°$$



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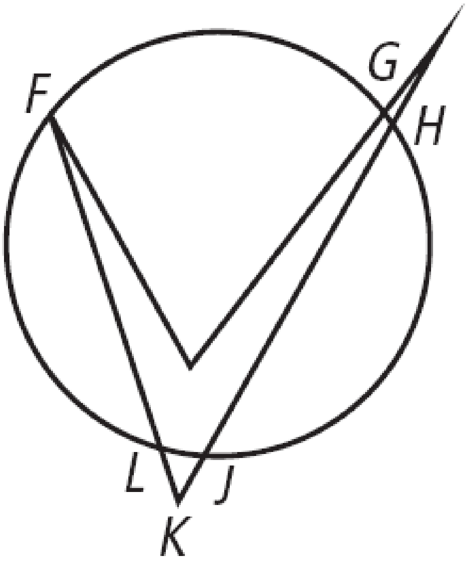
Example 4: Design Application

In the company logo shown, $m\overset{\frown}{FH} = 108°$, and $m\overset{\frown}{LJ} = 12°$. What is $m\angle FKH$?



$$m\angle FKH = \frac{1}{2}\left(m\overset{\frown}{FH} - m\overset{\frown}{LJ}\right)$$

$$= \frac{1}{2}\left(108° - 12°\right)$$

$$= \frac{1}{2}\left(96°\right) = 48°$$

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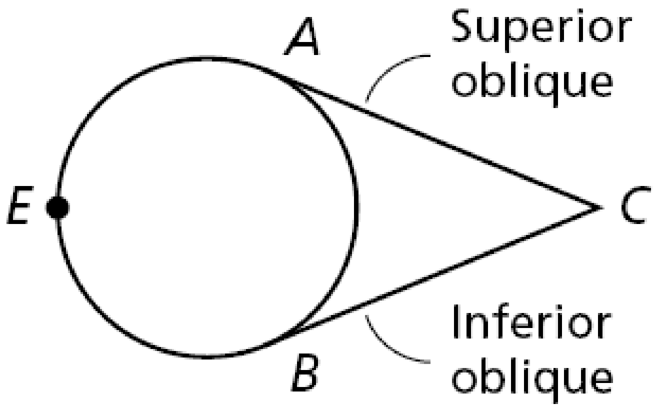
Check It Out! Example 4

Two of the six muscles that control eye movement are attached to the eyeball and intersect behind the eye. If $m\overset{\frown}{AEB} = 225°$, what is $m\angle ACB$?

$$m\angle ACB = \frac{1}{2}\left(m\overset{\frown}{AEB} - m\overset{\frown}{AB}\right)$$

$$= \frac{1}{2}\left(225° - 135°\right)$$

$$= \frac{1}{2}\left(90°\right) = 45°$$



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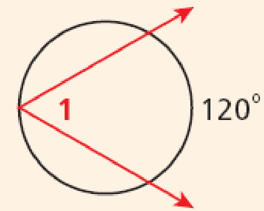
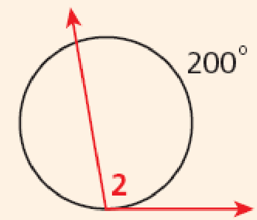
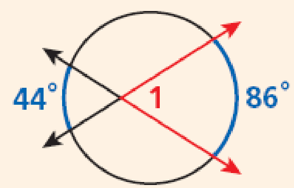
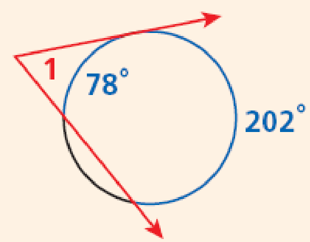
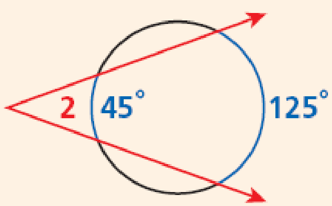
Angle Relationships in Circles

VERTEX OF THE ANGLE	MEASURE OF ANGLE	DIAGRAMS	
On a circle	Half the measure of its intercepted arc	 $m\angle 1 = 60°$	 $m\angle 2 = 100°$
Inside a circle	Half the sum of the measures of its intercepted arcs		$m\angle 1 = \frac{1}{2}(44° + 86°)$ $= 65°$
Outside a circle	Half the difference of the measures of its intercepted arcs	 $m\angle 1 = \frac{1}{2}(202° - 78°)$ $= 62°$	 $m\angle 2 = \frac{1}{2}(125° - 45°)$ $= 40°$

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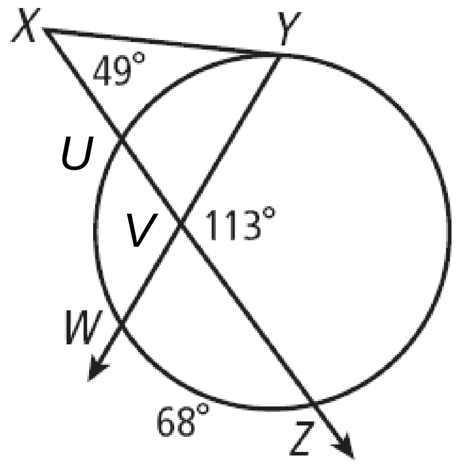
Example 5: Finding Arc Measures

Find $m\overset{\frown}{YZ}$.



Step 1 Find $m\overset{\frown}{UY}$.

$$m\angle XVY = \frac{1}{2}\left(m\overset{\frown}{UY} + m\overset{\frown}{WZ}\right)$$

If a tangent and a secant intersect on a ⊙ at the pt. of tangency, then the measure of the ∠ formed is half the measure of its intercepted arc.

$$180° - 113° = \frac{1}{2}\left(m\overset{\frown}{UY} + 68°\right)$$

Substitute 180 – 113 for m∠XVY and 68 for $m\overset{\frown}{WZ}$

$$134° = m\overset{\frown}{UY} + 68°$$

Multiply both sides by 2.

$$m\overset{\frown}{UY} = 66°$$

Subtract 68 from both sides.

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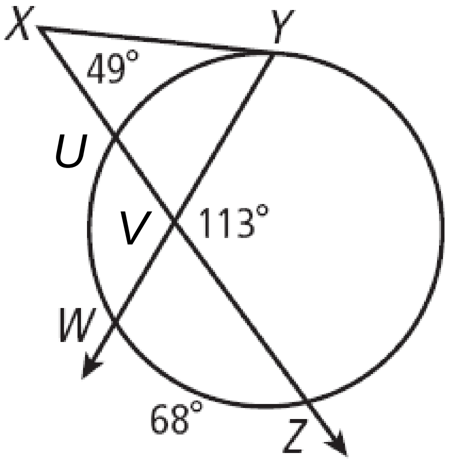
Example 5 Continued



Step 2 Find $m\overset{\frown}{YZ}$.

$m\angle X = \frac{1}{2}\left(m\overset{\frown}{YZ} - m\overset{\frown}{UY}\right)$ *Thm. 11-5-3*

$49° = \frac{1}{2}\left(m\overset{\frown}{YZ} - 66°\right)$ *Substitute the given values.*

$98° = m\overset{\frown}{YZ} - 66°$ *Multiply both sides by 2.*

$164° = m\overset{\frown}{YZ}$ *Add 66 to both sides.*

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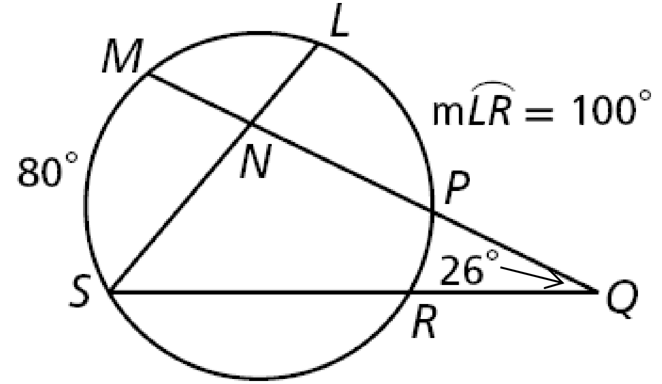
Check It Out! Example 5

Find m$\overset{\frown}{LP}$



Step 1 Find m$\overset{\frown}{PR}$.

$$m\angle PQR = \frac{1}{2}\left(m\overset{\frown}{MS} - m\overset{\frown}{PR}\right)$$

$$26° = \frac{1}{2}\left(80° - m\overset{\frown}{PR}\right)$$

$$52° = 80° - m\overset{\frown}{PR}$$

$$28° = m\overset{\frown}{PR}$$

Step 2 Find m$\overset{\frown}{LP}$.

$$m\overset{\frown}{LR} = m\overset{\frown}{LP} + m\overset{\frown}{PR}$$

$$100° = m\overset{\frown}{LP} + 28°$$

$$72° = m\overset{\frown}{LP}$$

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