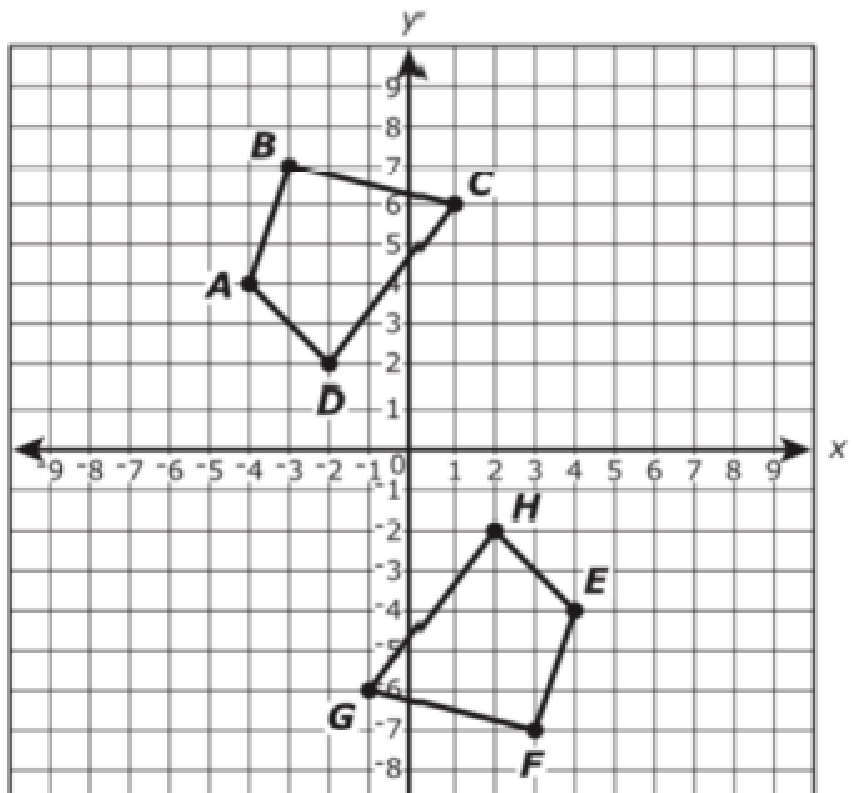
12-2 Arcs and Chords

Warm Up

Lesson Presentation

Lesson Quiz

Quadrilaterals *ABCD* and *EFGH* are shown in the coordinate plane.



Quadrilateral *EFGH* is the image of *ABCD* after a transformation or sequence of transformations.

Which could be the transformation or sequence of transformations?

Select all that apply.

Ⓐ a translation of 3 units to the right, followed by a reflection across the x-axis
Ⓑ a rotation of 180°
Ⓒ a translation of 12 units downward, followed by a reflection across the y-axis
Ⓓ a reflection across the y-axis, followed by a reflection across the x-axis
Ⓔ a reflection across the line with equation y = x

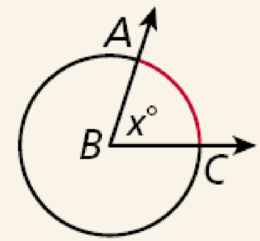
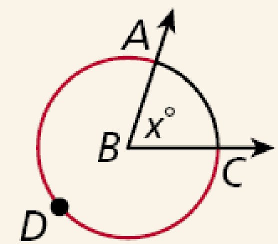
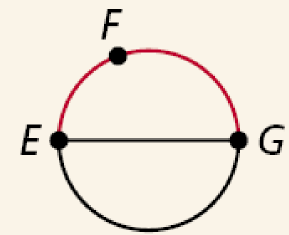
Objectives

Apply properties of arcs.

Apply properties of chords.

A **central angle** is an angle whose vertex is the center of a circle. An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

Arcs and Their Measure

ARC	MEASURE	DIAGRAM
A **minor arc** is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\widehat{AC} = m\angle ABC = x°$	
A **major arc** is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\widehat{ADC} = 360° - m\angle ABC$ $= 360° - x°$	
If the endpoints of an arc lie on a diameter, the arc is a **semicircle**.	The measure of a semicircle is equal to 180°. $m\widehat{EFG} = 180°$	

Example 1: Data Application

The circle graph shows the types of grass planted in the yards of one neighborhood. Find $m\overset{\frown}{KLF}$.

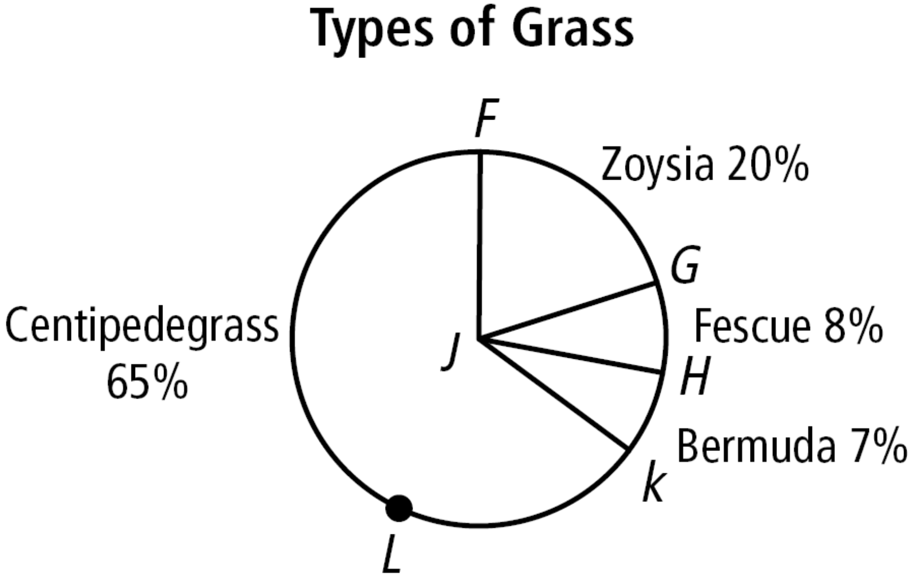
$m\overset{\frown}{KLF} = 360° – m\angle KJF$

$m\angle KJF = 0.35(360°)$

$= 126°$

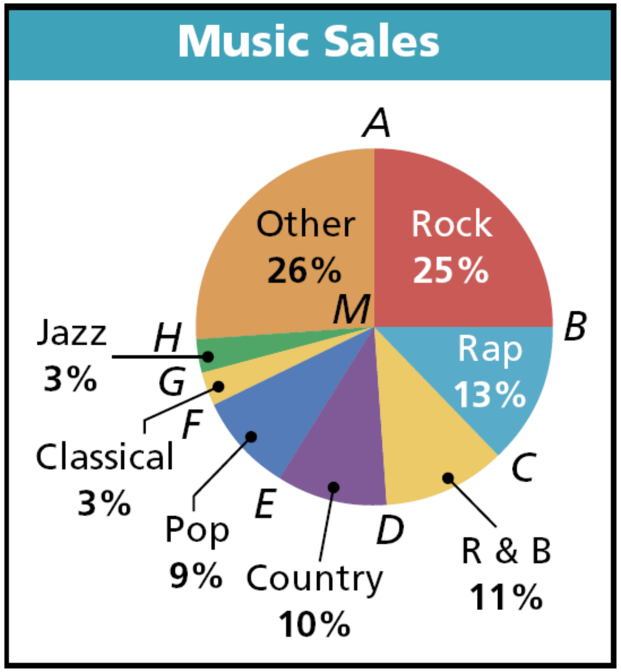
$m\overset{\frown}{KLF} = 360° – 126°$

$= 234°$



Check It Out! Example 1

Use the graph to find each of the following.



a. $m\angle FMC$

$m\angle FMC = 0.30(360°)$

$= 108°$

Central ∠ is 30% of the ⊙.

b. $m\widehat{AHB} = 360° - m\angle AMB$

$m\angle AHB = 360° - 0.25(360°)$

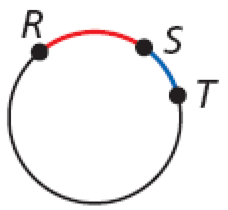
$= 270°$

c. $m\angle EMD = 0.10(360°)$

$= 36°$

Central ∠ is 10% of the ⊙.

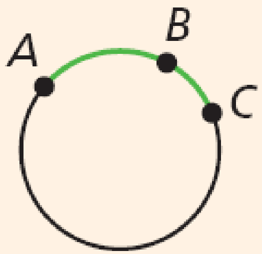
Adjacent arcs are arcs of the same circle that intersect at exactly one point. $\overset{\frown}{RS}$ and $\overset{\frown}{ST}$ are adjacent arcs.



Postulate 11-2-1 **Arc Addition Postulate**

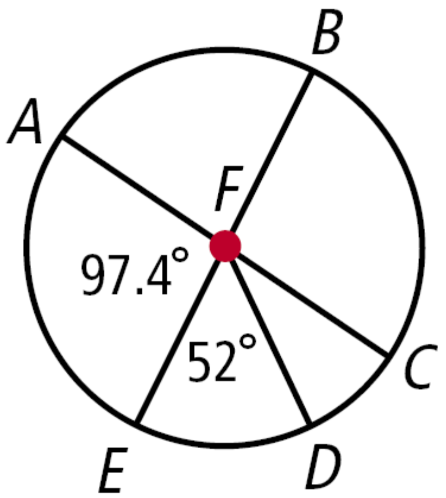
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$



Example 2: Using the Arc Addition Postulate

Find $m\overset{\frown}{BD}$.



$m\overset{\frown}{BC} = 97.4°$ *Vert. ∠s Thm.*

$m\angle CFD = 180° - (97.4° + 52°)$

$= 30.6°$ *Δ Sum Thm.*

$m\overset{\frown}{CD} = 30.6°$ *m∠CFD = 30.6°*

$m\overset{\frown}{BD} = m\overset{\frown}{BC} + m\overset{\frown}{CD}$ *Arc Add. Post.*

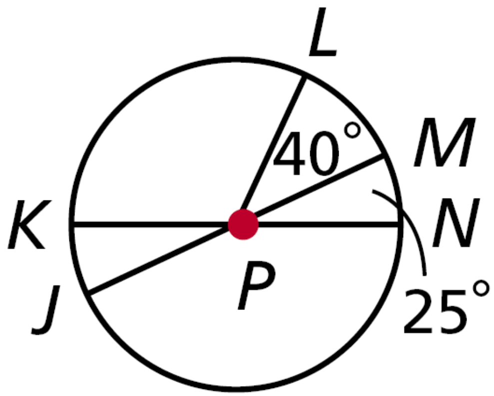
$= 97.4° + 30.6°$ *Substitute.*

$= 128°$ *Simplify.*

Check It Out! Example 2a

Find each measure.

$m\overset{\frown}{JKL}$



$m\angle KPL = 180° - (40 + 25)°$

$m\overset{\frown}{KL} = 115°$

$m\overset{\frown}{JKL} = m\overset{\frown}{JK} + m\overset{\frown}{KL}$ *Arc Add. Post.*

$= 25° + 115°$ *Substitute.*

$= 140°$ *Simplify.*

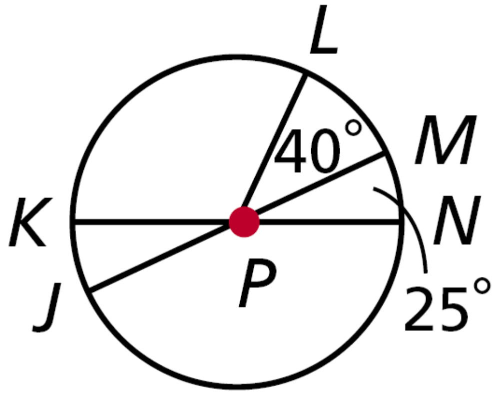
Check It Out! Example 2b

Find each measure.

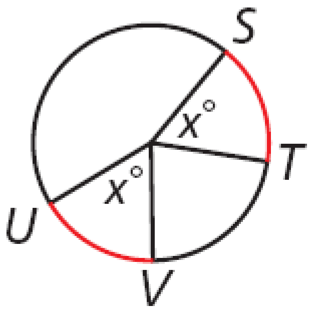
$m\overset{\frown}{LJN}$

$m\overset{\frown}{LJN} = 360° - (40 + 25)°$

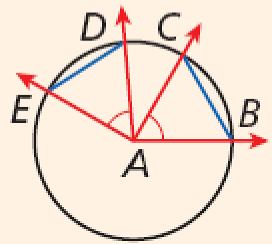
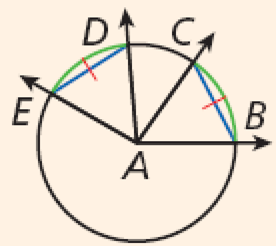
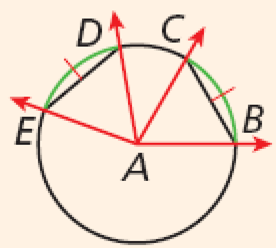
$= 295°$



Within a circle or congruent circles, **congruent arcs** are two arcs that have the same measure. In the figure $\overset{\frown}{ST} \cong \overset{\frown}{UV}$.

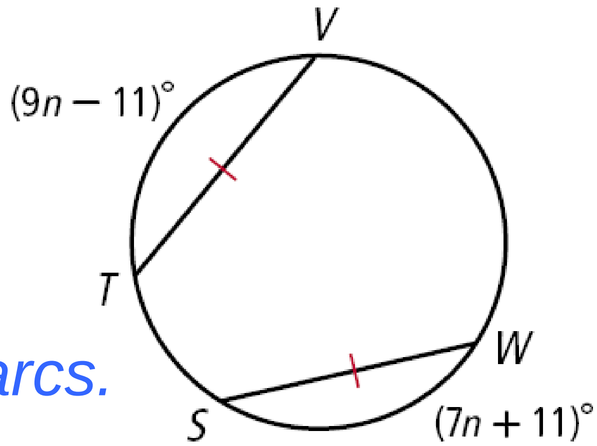


Theorem 11-2-2

THEOREM	HYPOTHESIS		CONCLUSION
In a circle or congruent circles:			
(1) Congruent central angles have congruent chords.		$\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.		$\overline{ED} \cong \overline{BC}$	$\overset{\frown}{DE} \cong \overset{\frown}{BC}$
(3) Congruent arcs have congruent central angles.		$\overset{\frown}{ED} \cong \overset{\frown}{BC}$	$\angle DAE \cong \angle BAC$

Example 3A: Applying Congruent Angles, Arcs, and Chords

$\overline{TV} \cong \overline{WS}$. Find $m\overset{\frown}{WS}$.



$\overset{\frown}{TV} \cong \overset{\frown}{WS}$	≅ chords have ≅ arcs.
$m\overset{\frown}{TV} = m\overset{\frown}{WS}$	Def. of ≅ arcs
$9n - 11 = 7n + 11$	Substitute the given measures.
$2n = 22$	Subtract 7n and add 11 to both sides.
$n = 11$	Divide both sides by 2.
$m\overset{\frown}{WS} = 7(11) + 11$	Substitute 11 for n.
$= 88°$	Simplify.

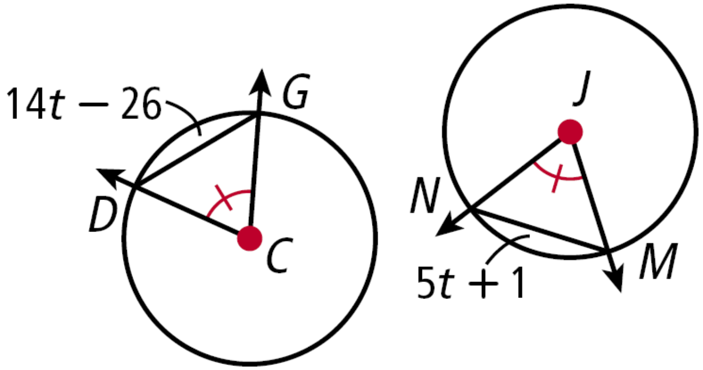
Example 3B: Applying Congruent Angles, Arcs, and Chords

⊙*C* ≅ ⊙*J*, and m∠*GCD* ≅ m∠*NJM*. Find *NM*.

$\overset{\frown}{GD} \cong \overset{\frown}{NM}$ — ∠GCD ≅ ∠NJM

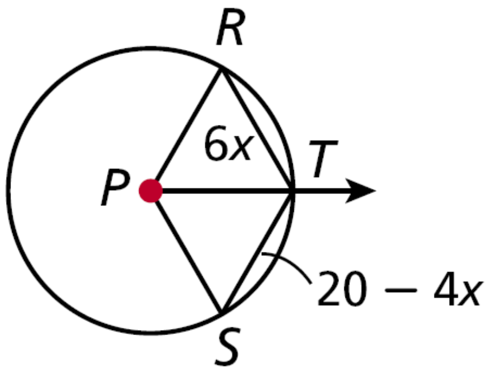
$\overline{GD} \cong \overline{NM}$ — ≅ arcs have ≅ chords.

$GD = NM$ — Def. of ≅ chords



Check It Out! Example 3a

$\overrightarrow{PT}$ **bisects** $\angle RPS$**. Find** RT**.**



$\angle RPT \cong \angle SPT$

$m\overset{\frown}{RT} \cong m\overset{\frown}{TS}$

$RT = TS$

$6x = 20 - 4x$

$10x = 20$ *Add 4x to both sides.*

$x = 2$ *Divide both sides by 10.*

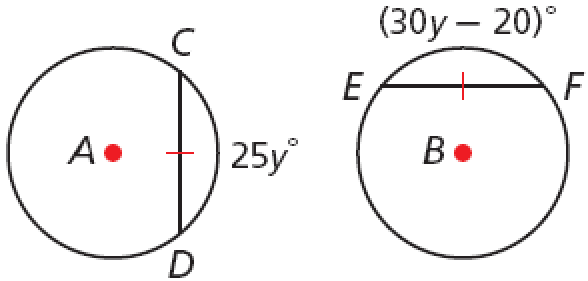
$RT = 6(2)$ *Substitute 2 for x.*

$RT = 12$ *Simplify.*

Check It Out! Example 3b

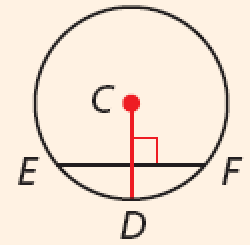
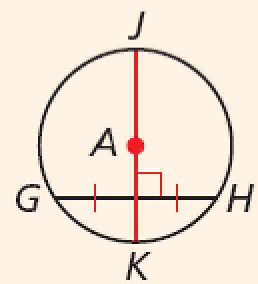
Find each measure.

$\odot A \cong \odot B$, and $\overline{CD} \cong \overline{EF}$. Find $m\overset{\frown}{CD}$.

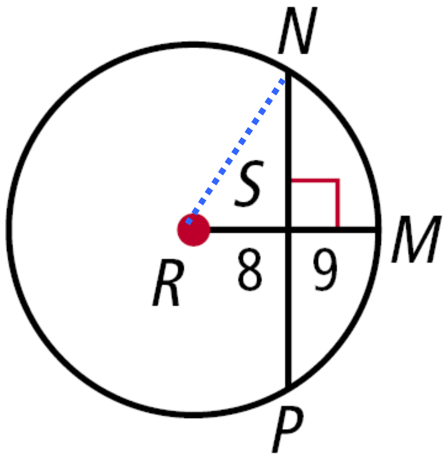


$m\overset{\frown}{CD} = m\overset{\frown}{EF}$	*≅ chords have ≅ arcs.*
$25y° = (30y – 20)°$	*Substitute.*
$20 = 5y$	*Subtract 25y from both sides. Add 20 to both sides.*
$4 = y$	*Divide both sides by 5.*
$CD = 25(4)$	*Substitute 4 for y.*
$m\overset{\frown}{CD} = 100°$	*Simplify.*

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
11-2-3 In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.	 $\overline{CD} \perp \overline{EF}$	$\overline{CD}$ bisects $\overline{EF}$ and $\widehat{EF}$.
11-2-4 In a circle, the perpendicular bisector of a chord is a radius (or diameter).	 $\overline{JK}$ is $\perp$ bisector of $\overline{GH}$.	$\overline{JK}$ is a diameter of $\odot A$.

Example 4: Using Radii and Chords



Find *NP*.

Step 1 Draw radius $\overline{RN}$.

$RN = 17$ *Radii of a ⊙ are ≅.*

Step 2 Use the Pythagorean Theorem.

$SN^2 + RS^2 = RN^2$

$SN^2 + 8^2 = 17^2$ *Substitute 8 for RS and 17 for RN.*

$SN^2 = 225$ *Subtract 8^2 from both sides.*

$SN = 15$ *Take the square root of both sides.*

Step 3 Find *NP*.

$NP = 2(15) = 30$ *$\overline{RM} \perp \overline{NP}$, so $\overline{RM}$ bisects $\overline{NP}$.*