12-1 Lines That Intersect Circles

Warm Up

Lesson Presentation

Lesson Quiz

The table shows the approximate measurements of the Great Pyramid of Giza in Egypt and the Pyramid of Kukulcan in Mexico.

Pyramid	Height (meters)	Area of Base (square meters)
Great Pyramid of Giza	147	52,900
Pyramid of Kukulcan	30	3,025

Approximately what is the difference between the volume of the Great Pyramid of Giza and the volume of the Pyramid of Kukulcan?

Ⓐ 1, 945, 000 cubic meters
Ⓑ 2, 562, 000 cubic meters
Ⓒ 5, 835, 000 cubic meters
Ⓓ 7, 686, 000 cubic meters

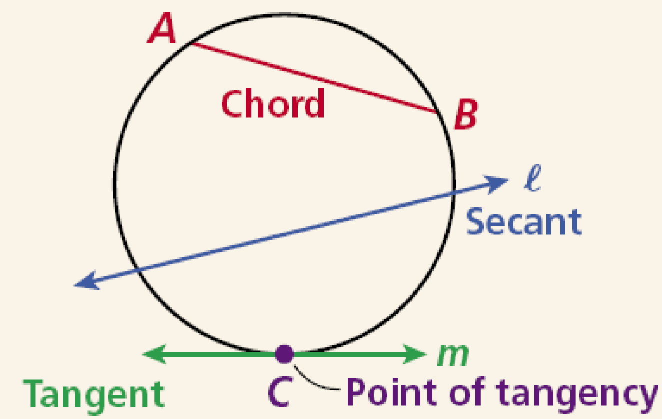
Objectives

Identify tangents, secants, and chords.

Use properties of tangents to solve problems.

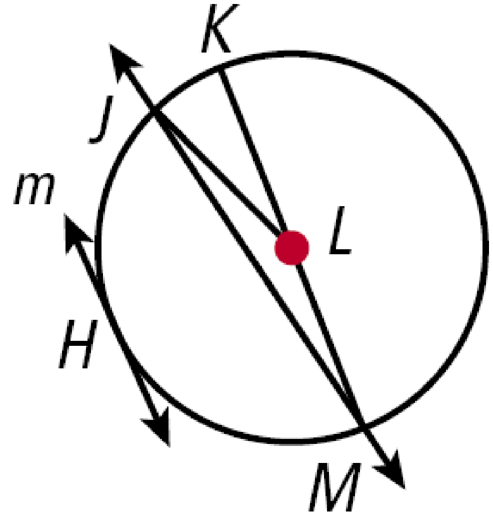
The **interior of a circle** is the set of all points inside the circle. The **exterior of a circle** is the set of all points outside the circle.

Lines and Segments That Intersect Circles

TERM	DIAGRAM
A **chord** is a segment whose endpoints lie on a circle.	
A **secant** is a line that intersects a circle at two points.	
A **tangent** is a line in the same plane as a circle that intersects it at exactly one point.	
The point where the tangent and a circle intersect is called the **point of tangency**.	

Example 1: Identifying Lines and Segments That Intersect Circles

Identify each line or segment that intersects ⊙*L*.



chords: $\overline{JM}$ and $\overline{KM}$

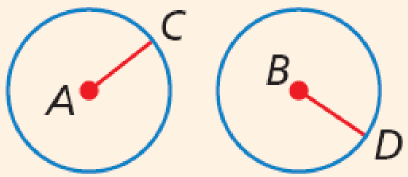
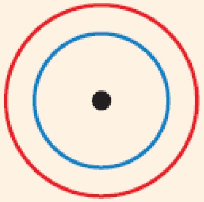
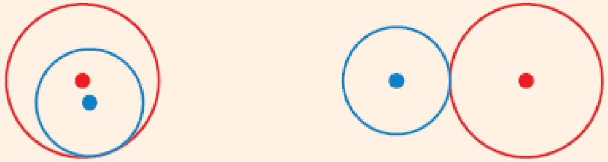
secant: $\overleftrightarrow{JM}$

tangent: *m*

diameter: $\overline{KM}$

radii: $\overline{LK}$, $\overline{LJ}$, and $\overline{LM}$

Pairs of Circles

TERM	DIAGRAM
Two circles are **congruent circles** if and only if they have congruent radii.	 $\odot A \cong \odot B$ if $\overline{AC} \cong \overline{BD}$. $\overline{AC} \cong \overline{BD}$ if $\odot A \cong \odot B$.
Concentric circles are coplanar circles with the same center.	
Two coplanar circles that intersect at exactly one point are called **tangent circles**.	 Internally tangent circles Externally tangent circles

Example 2: Identifying Tangents of Circles

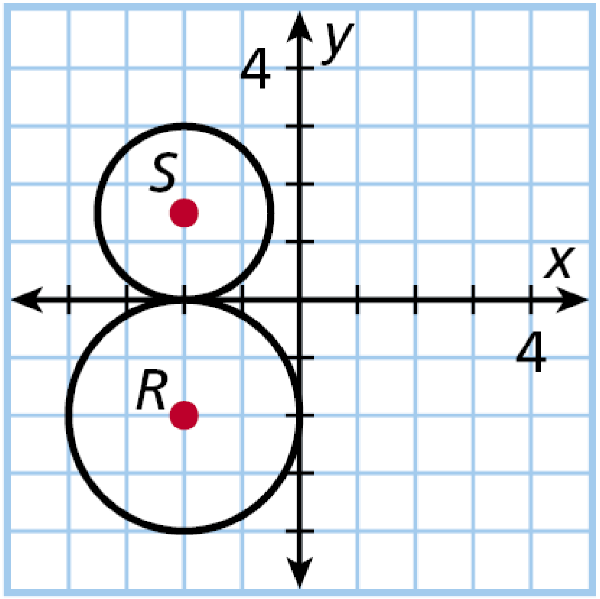
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of ⊙*R*: 2

Center is (–2, –2). Point on ⊙ is (–2,0). Distance between the 2 points is 2.

radius of ⊙*S*: 1.5

Center is (–2, 1.5). Point on ⊙ is (–2,0). Distance between the 2 points is 1.5.



Example 2 Continued

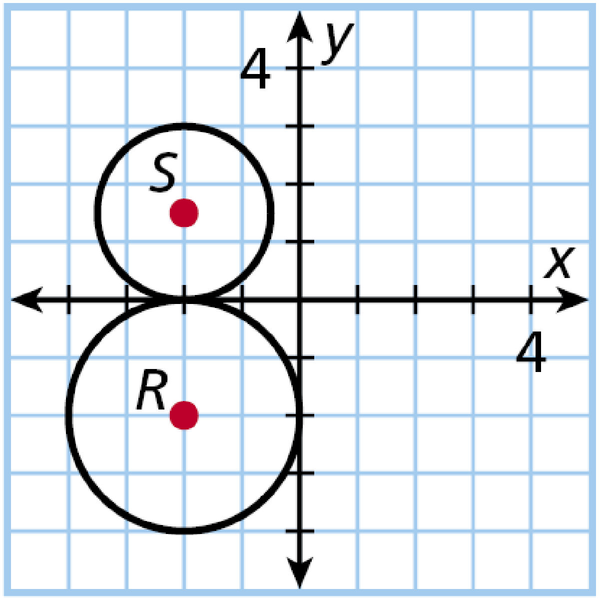
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

point of tangency: (–2, 0)

Point where the ⊙s and tangent line intersect

equation of tangent line: $y = 0$

Horizontal line through (–2,0)



Check It Out! Example 2

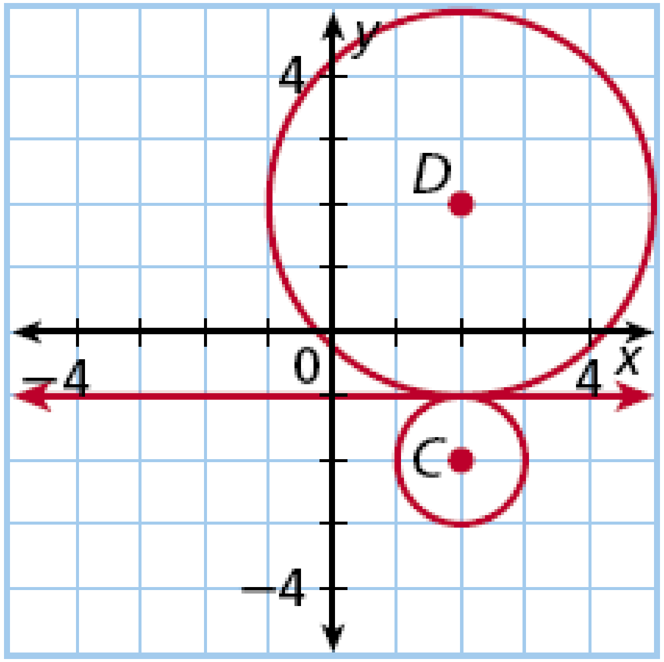
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of ⊙*C*: 1

Center is (2, –2). Point on ⊙ is (2, –1). Distance between the 2 points is 1.

radius of ⊙*D*: 3

Center is (2, 2). Point on ⊙ is (2, –1). Distance between the 2 points is 3.



Check It Out! Example 2 Continued

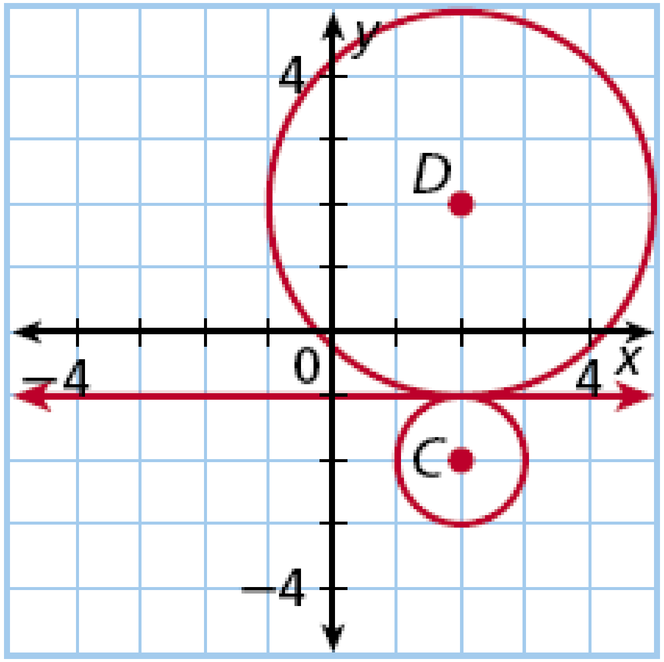
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

Pt. of tangency: **(**2, –1**)**

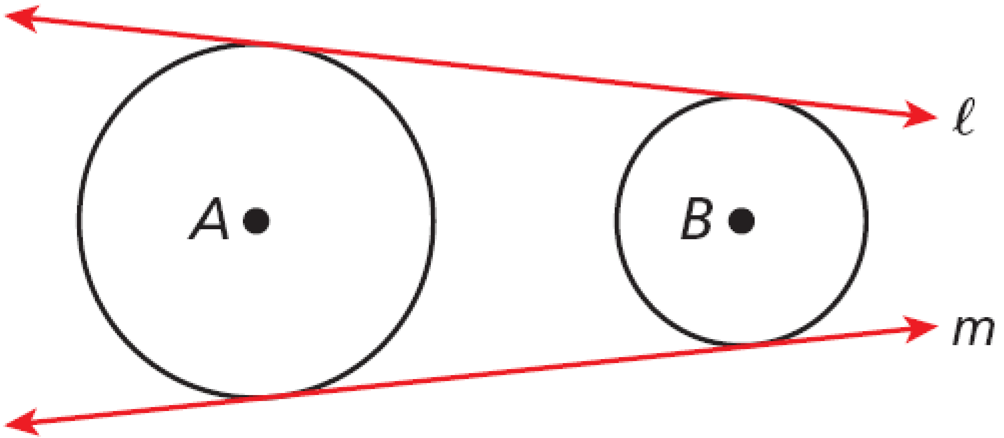
Point where the ⊙s and tangent line intersect

eqn. of tangent line: $y = -1$

Horizontal line through (2,-1)

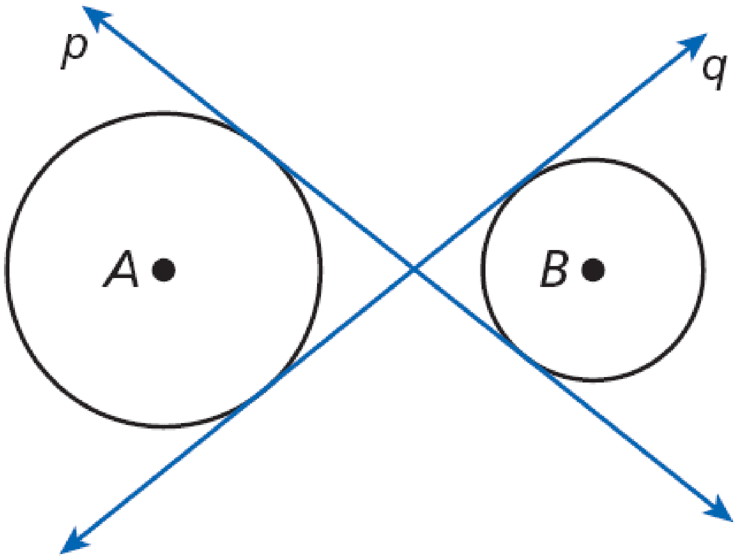


A **common tangent** is a line that is tangent to two circles.



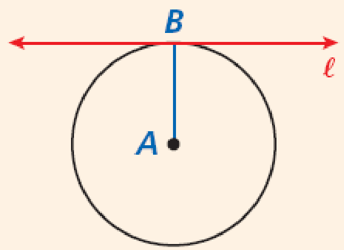
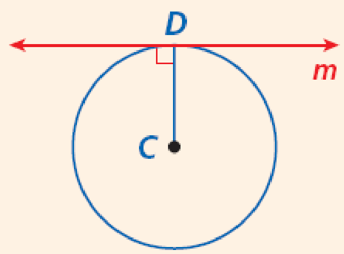
Lines ℓ and *m* are common external tangents to ⊙*A* and ⊙*B*.

A **common tangent** is a line that is tangent to two circles.

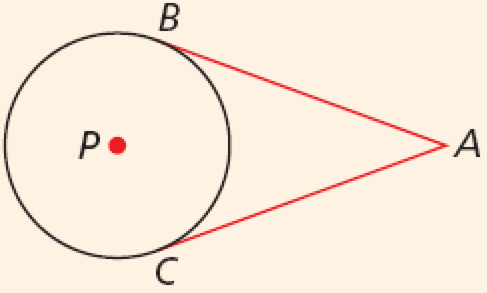


Lines *p* and *q* are common internal tangents to ⊙*A* and ⊙*B*.

Theorems

THEOREM	HYPOTHESIS	CONCLUSION
11-1-1 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (line tangent to ⊙ → line ⊥ to radius)	 ℓ is tangent to ⊙*A*	$\ell \perp \overline{AB}$
11-1-2 If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle. (line ⊥ to radius → line tangent to ⊙)	 *m* is ⊥ to $\overline{CD}$ at *D*	*m* is tangent to ⊙*C*.

Theorem 11-1-3

<table>
<tr><th>THEOREM</th><th>HYPOTHESIS</th><th>CONCLUSION</th></tr>
<tr><td>If two segments are tangent to a circle from the same external point, then the segments are congruent. (2 segs. tangent to ⊙ from same ext. pt. → segs. ≅)</td><td>
$\overline{AB}$ and $\overline{AC}$ are tangent to ⊙P.</td><td>$\overline{AB} \cong \overline{AC}$</td></tr>
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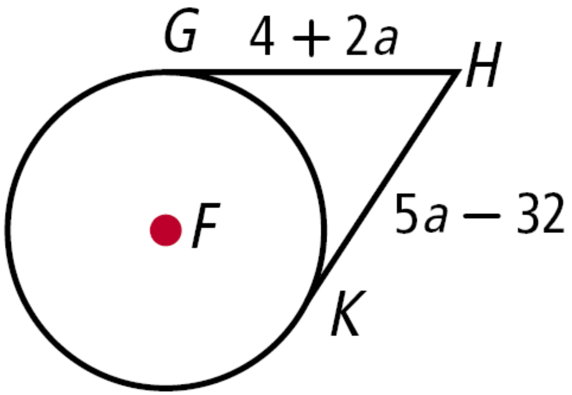
Example 4: Using Properties of Tangents

$\overline{HK}$ and $\overline{HG}$ are tangent to $\odot F$. Find HG.



$HK = HG$ — 2 segments tangent to ⊙ from same ext. point → segments ≅.

$5a - 32 = 4 + 2a$ — Substitute 5a – 32 for HK and 4 + 2a for HG.

$3a - 32 = 4$ — Subtract 2a from both sides.

$3a = 36$ — Add 32 to both sides.

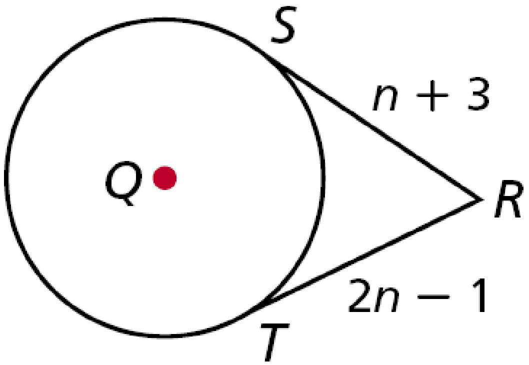
$a = 12$ — Divide both sides by 3.

$HG = 4 + 2(12)$ — Substitute 12 for a.

$= 28$ — Simplify.

Check It Out! Example 4b

$\overline{RS}$ and $\overline{RT}$ are tangent to ⊙*Q*. Find *RS*.



$RS = RT$ — *2 segments tangent to ⊙ from same ext. point → segments ≅.*

$n + 3 = 2n – 1$ — *Substitute n + 3 for RS and 2n – 1 for RT.*

$4 = n$ — *Simplify.*

$RS = 4 + 3$ — *Substitute 4 for n.*

$= 7$ — *Simplify.*