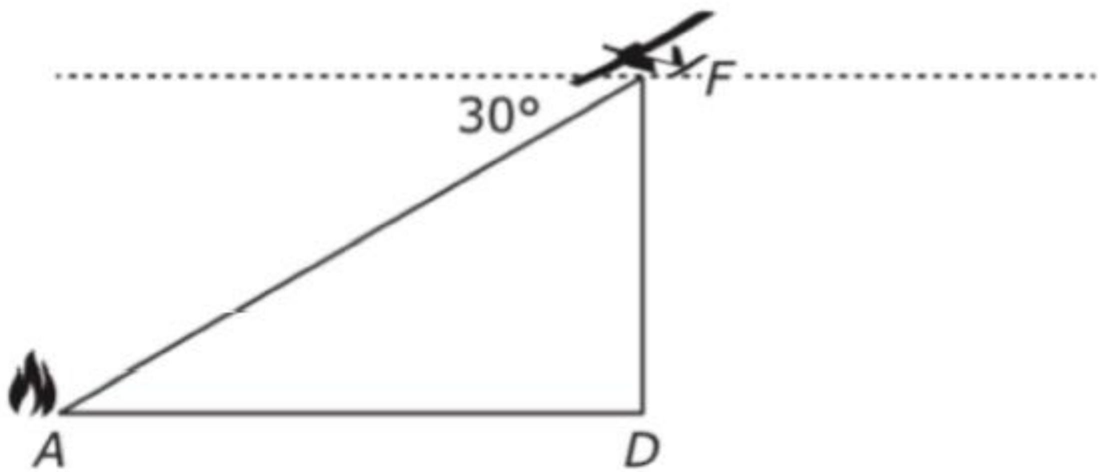
11-3 Volume of Pyramids and Cones

Warm Up

Lesson Presentation

Lesson Quiz

The UAV is flying at a speed of 13 meters per second in the direction toward the fire. Suppose the altitude of the UAV is now 800 meters. The new position is reprented at *F* in the figure.



From its position at point *F*, how many minutes, to the nearest tenth of a minute, will it take the UAV to be directly over the fire?

Ⓐ 0.6
Ⓑ 1.2
Ⓒ 1.8
Ⓓ 2.0

Objectives

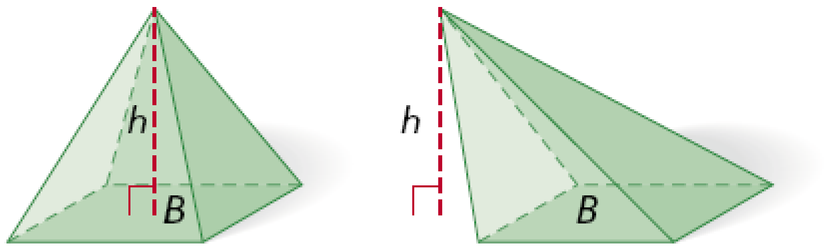
Learn and apply the formula for the volume of a pyramid.

Learn and apply the formula for the volume of a cone.

The square pyramids are congruent, so they have the same volume. The volume of each pyramid is one third the volume of the cube.

Volume of a Pyramid

The volume of a pyramid with base area B and height h is $V = \frac{1}{3}Bh$.



Example 1A: Finding Volumes of Pyramids

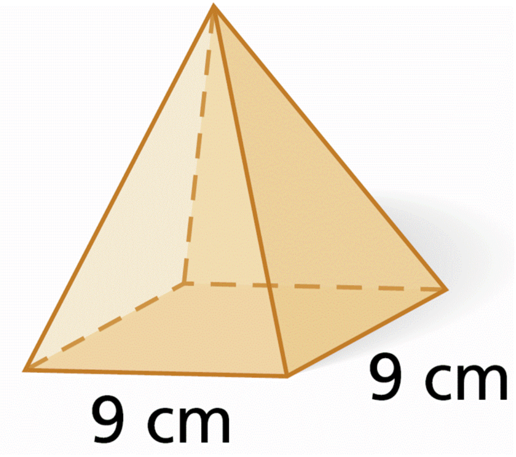
Find the volume a rectangular pyramid with length 11 m, width 18 m, and height 23 m.

$$V = \frac{1}{3}Bh = \frac{1}{3}(11 \bullet 18)(23) = 1518 \text{ m}^3$$

Example 1B: Finding Volumes of Pyramids

Find the volume of the square pyramid with base edge length 9 cm and height 14 cm.

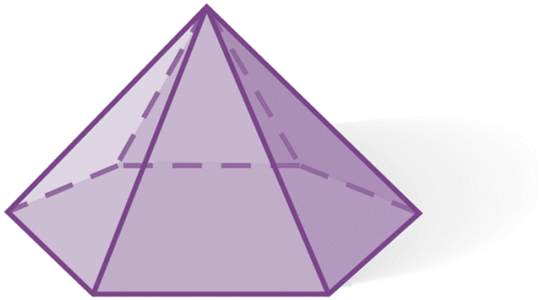
The base is a square with a side length of 9 cm, and the height is 14 cm.



$$V = \frac{1}{3}Bh = \frac{1}{3}(9^2)(14) = 378 \text{ cm}^3$$

Example 1C: Finding Volumes of Pyramids

Find the volume of the regular hexagonal pyramid with height equal to the apothem of the base



Step 1 Find the area of the base.

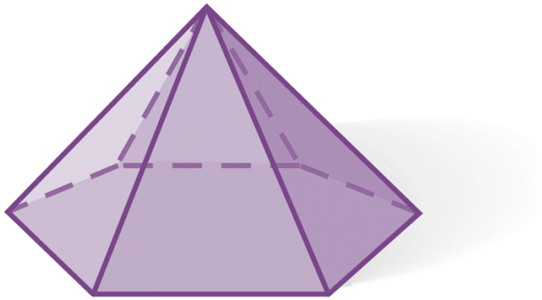
$$B = \frac{1}{2}aP$$ *Area of a regular polygon*

$$= \frac{1}{2}\left(6\sqrt{3}\right)\left(6(12)\right)$$ *Substitute $6\sqrt{3}$ for a and 6(12) for P.*

$$= 216\sqrt{3}\ \text{ft}^3$$ *Simplify.*

Example 1C Continued

Find the volume of the regular hexagonal pyramid with height equal to the apothem of the base



12 ft

Step 2 Use the base area and the height to find the volume. The height is equal to the apothem, $a = 6\sqrt{3}$ ft.

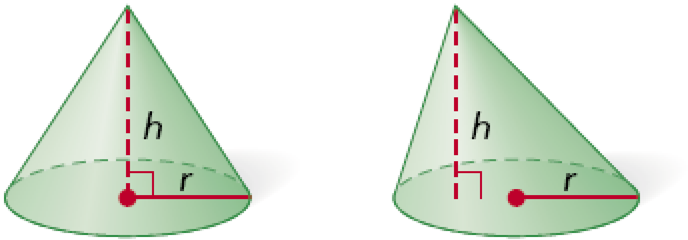
$V = \frac{1}{3}Bh$ *Volume of a pyramid.*

$= \frac{1}{3}\left(216\sqrt{3}\right)\left(6\sqrt{3}\right)$ *Substitute $216\sqrt{3}$ for B and $6\sqrt{3}$ for h.*

$= 1296$ ft^3 *Simplify.*

Volume of Cones

The volume of a cone with base area B, radius r, and height h is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$.



Example 3A: Finding Volumes of Cones

Find the volume of a cone with radius 7 cm and height 15 cm. Give your answers both in terms of π and rounded to the nearest tenth.

$V = \frac{1}{3}\pi r^2 h$ *Volume of a pyramid*

$= \frac{1}{3}\pi (7)^2 (15)$ *Substitute 7 for r and 15 for h.*

$= 245\pi \text{ cm}^3 \approx 769.7 \text{ cm}^3$ *Simplify.*

Example 3B: Finding Volumes of Cones

Find the volume of a cone with base circumference 25π in. and a height 2 in. more than twice the radius.

Step 1 Use the circumference to find the radius.

$2\pi r = 25\pi$ *Substitute 25π for the circumference.*

$r = 12.5$ *Solve for r.*

Step 2 Use the radius to find the height.

$h = 2(12.5) + 2 = 27$ in. *The height is 2 in. more than twice the radius.*

Example 3B Continued

Find the volume of a cone with base circumference 25π in. and a height 2 in. more than twice the radius.

Step 3 Use the radius and height to find the volume.

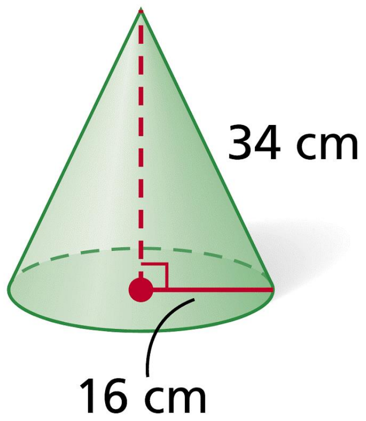
$$V = \frac{1}{3}\pi r^2 h$$ *Volume of a pyramid.*

$$= \frac{1}{3}\pi(12.5)^2(27)$$ *Substitute 12.5 for r and 27 for h.*

$$= 1406.25\pi \text{ in}^3 \approx 4417.9 \text{ in}^3$$ *Simplify.*

Example 3C: Finding Volumes of Cones

Find the volume of a cone.



Step 1 Use the Pythagorean Theorem to find the height.

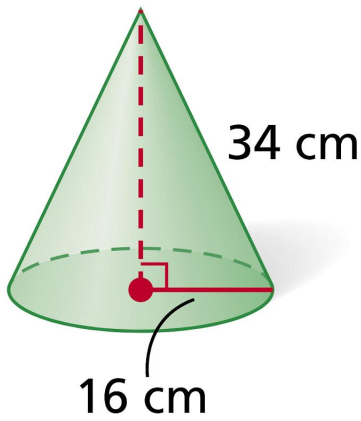
$16^2 + h^2 = 34^2$ *Pythagorean Theorem*

$h^2 = 900$ *Subtract 16^2 from both sides.*

$h = 30$ *Take the square root of both sides.*

Example 3C Continued

Find the volume of a cone.



Step 2 Use the radius and height to find the volume.

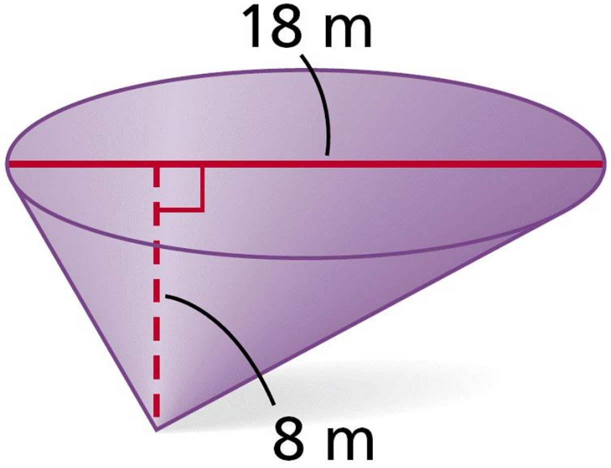
$$V = \frac{1}{3}\pi r^2 h$$ *Volume of a cone*

$$= \frac{1}{3}\pi(16)^2(30)$$ *Substitute 16 for r and 30 for h.*

$$\approx 2560\pi \text{ cm}^3 \approx 8042.5 \text{ cm}^3$$ *Simplify.*

Check It Out! Example 3

Find the volume of the cone.



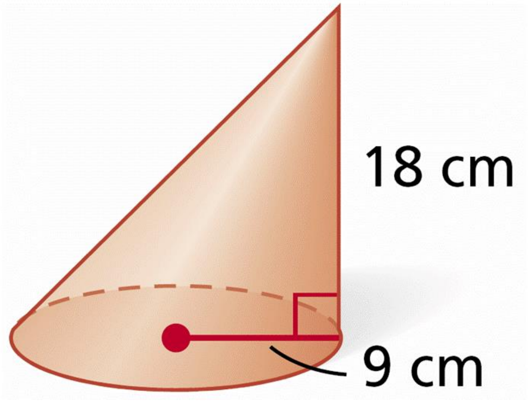
$V = \frac{1}{3}\pi r^2 h$ *Volume of a cone*

$= \frac{1}{3}\pi(9)^2(8)$ *Substitute 9 for r and 8 for h.*

$\approx 216\pi \text{ m}^3 \approx 678.6 \text{ m}^3$ *Simplify.*

Check It Out! Example 4

The radius and height of the cone are doubled. Describe the effect on the volume.



original dimensions:

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(9)^2(18) = 486\pi \text{ cm}^3$$

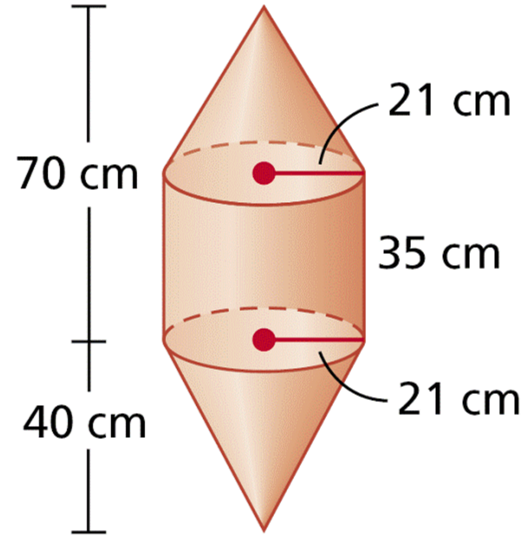
radius and height doubled:

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(18)^2(36) = 3888\pi \text{ cm}^3$$

The volume is multiplied by 8.

Example 5: Finding Volumes of Composite Three-Dimensional Figures

Find the volume of the composite figure. Round to the nearest tenth.



The volume of the upper cone is

$$V_{upper} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(21)^2(70 - 35) = 5145\pi \text{ cm}^3.$$